C4 VECTORS — Worksheet C

1 Sketch each line on a separate diagram given its vector equation.

a $\mathbf{r} = 2\mathbf{i} + s\mathbf{j}$ **b** $\mathbf{r} = s(\mathbf{i} + \mathbf{j})$ **c** $\mathbf{r} = \mathbf{i} + 4\mathbf{j} + s(\mathbf{i} + 2\mathbf{j})$

d $\mathbf{r} = 3\mathbf{j} + s(3\mathbf{i} - \mathbf{j})$ **e** $\mathbf{r} = -4\mathbf{i} + 2\mathbf{j} + s(2\mathbf{i} - \mathbf{j})$ **f** $\mathbf{r} = (2s + 1)\mathbf{i} + (3s - 2)\mathbf{j}$

2 Write down a vector equation of the straight line

a parallel to the vector $(3\mathbf{i} - 2\mathbf{j})$ which passes through the point with position vector $(-\mathbf{i} + \mathbf{j})$,

b parallel to the x-axis which passes through the point with coordinates (0, 4),

c parallel to the line $\mathbf{r} = 2\mathbf{i} + t(\mathbf{i} + 5\mathbf{j})$ which passes through the point with coordinates (3, –1).

3 Find a vector equation of the straight line which passes through the points with position vectors

a $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ **b** $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ **c** $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

4 Find the value of the constant c such that line with vector equation $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + \lambda(c\mathbf{i} + 2\mathbf{j})$

a passes through the point (0, 5),

b is parallel to the line $\mathbf{r} = -2\mathbf{i} + 4\mathbf{j} + \mu(6\mathbf{i} + 3\mathbf{j})$.

5 Find a vector equation for each line given its cartesian equation.

a $x = -1$ **b** $y = 2x$ **c** $y = 3x + 1$

d $y = \frac{3}{4}x - 2$ **e** $y = 5 - \frac{1}{2}x$ **f** $x - 4y + 8 = 0$

6 A line has the vector equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \lambda(3\mathbf{i} + 2\mathbf{j})$.

a Write down parametric equations for the line.

b Hence find the cartesian equation of the line in the form $ax + by + c = 0$, where a, b and c are integers.

7 Find a cartesian equation for each line in the form $ax + by + c = 0$, where a, b and c are integers.

a $\mathbf{r} = 3\mathbf{i} + \lambda(\mathbf{i} + 2\mathbf{j})$ **b** $\mathbf{r} = \mathbf{i} + 4\mathbf{j} + \lambda(3\mathbf{i} + \mathbf{j})$ **c** $\mathbf{r} = 2\mathbf{j} + \lambda(4\mathbf{i} - \mathbf{j})$

d $\mathbf{r} = -2\mathbf{i} + \mathbf{j} + \lambda(5\mathbf{i} + 2\mathbf{j})$ **e** $\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + \lambda(-3\mathbf{i} + 4\mathbf{j})$ **f** $\mathbf{r} = (\lambda + 3)\mathbf{i} + (-2\lambda - 1)\mathbf{j}$

8 For each pair of lines, determine with reasons whether they are identical, parallel but not identical or not parallel.

a $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + s\begin{pmatrix} 3 \\ -1 \end{pmatrix}$, $\mathbf{r} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -6 \\ 2 \end{pmatrix}$

b $\mathbf{r} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + s\begin{pmatrix} 1 \\ 4 \end{pmatrix}$, $\mathbf{r} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} + t\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

c $\mathbf{r} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} + s\begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $\mathbf{r} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + t\begin{pmatrix} 3 \\ 6 \end{pmatrix}$

9 Find the position vector of the point of intersection of each pair of lines.

a $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda\mathbf{i}$, $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mu(3\mathbf{i} + \mathbf{j})$

b $\mathbf{r} = 4\mathbf{i} + \mathbf{j} + \lambda(-\mathbf{i} + \mathbf{j})$, $\mathbf{r} = 5\mathbf{i} - 2\mathbf{j} + \mu(2\mathbf{i} - 3\mathbf{j})$

c $\mathbf{r} = \mathbf{j} + \lambda(2\mathbf{i} - \mathbf{j})$, $\mathbf{r} = 2\mathbf{i} + 10\mathbf{j} + \mu(-\mathbf{i} + 3\mathbf{j})$

d $\mathbf{r} = -\mathbf{i} + 5\mathbf{j} + \lambda(-4\mathbf{i} + 6\mathbf{j})$, $\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + \mu(-\mathbf{i} + 2\mathbf{j})$

e $\mathbf{r} = -2\mathbf{i} + 11\mathbf{j} + \lambda(-3\mathbf{i} + 4\mathbf{j})$, $\mathbf{r} = -3\mathbf{i} - 7\mathbf{j} + \mu(5\mathbf{i} + 3\mathbf{j})$

f $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(3\mathbf{i} + 2\mathbf{j})$, $\mathbf{r} = 3\mathbf{i} + 5\mathbf{j} + \mu(\mathbf{i} + 4\mathbf{j})$

10 Write down a vector equation of the straight line

a parallel to the vector $(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$ which passes through the point with position vector $(4\mathbf{i} + \mathbf{k})$,

b perpendicular to the xy-plane which passes through the point with coordinates (2, 1, 0),

c parallel to the line $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + t(2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$ which passes through the point with coordinates (−1, 4, 2).

11 The points A and B have position vectors $(5\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ and $(6\mathbf{i} - 3\mathbf{j} + \mathbf{k})$ respectively.

a Find $\overrightarrow{AB}$ in terms of **i**, **j** and **k**.

b Write down a vector equation of the straight line l which passes through A and B.

c Show that l passes through the point with coordinates (3, 9, −8).

12 Find a vector equation of the straight line which passes through the points with position vectors

a $(\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$ and $(5\mathbf{i} + 4\mathbf{j} + 6\mathbf{k})$

b $(3\mathbf{i} - 2\mathbf{k})$ and $(\mathbf{i} + 5\mathbf{j} + 2\mathbf{k})$

c $\mathbf{0}$ and $(6\mathbf{i} - \mathbf{j} + 2\mathbf{k})$

d $(-\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ and $(4\mathbf{i} - 7\mathbf{j} + \mathbf{k})$

13 Find the value of the constants a and b such that line $\mathbf{r} = 3\mathbf{i} - 5\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + a\mathbf{j} + b\mathbf{k})$

a passes through the point (9, −2, −8),

b is parallel to the line $\mathbf{r} = 4\mathbf{j} - 2\mathbf{k} + \mu(8\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$.

14 Find cartesian equations for each of the following lines.

a $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$

b $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 6 \\ 3 \end{pmatrix}$

c $\mathbf{r} = \begin{pmatrix} -1 \\ 5 \\ -2 \end{pmatrix} + \lambda\begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix}$

15 Find a vector equation for each line given its cartesian equations.

a $\frac{x-1}{3} = \frac{y+4}{2} = z - 5$

b $\frac{x}{4} = \frac{y-1}{-2} = \frac{z+7}{3}$

c $\frac{x+5}{-4} = y + 3 = z$

16 Show that the lines with vector equations $\mathbf{r} = 4\mathbf{i} + 3\mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ and $\mathbf{r} = 7\mathbf{i} + 2\mathbf{j} - 5\mathbf{k} + t(-3\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ intersect, and find the coordinates of their point of intersection.

17 Show that the lines with vector equations $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 4\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ are skew.

18 For each pair of lines, find the position vector of their point of intersection or, if they do not intersect, state whether they are parallel or skew.

a $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} + \lambda\begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} + \mu\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

b $\mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 6 \\ -2 \\ -1 \end{pmatrix} + \mu\begin{pmatrix} -4 \\ 2 \\ 6 \end{pmatrix}$

c $\mathbf{r} = \begin{pmatrix} 8 \\ 2 \\ -4 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -2 \\ 2 \\ 8 \end{pmatrix} + \mu\begin{pmatrix} 4 \\ -3 \\ -4 \end{pmatrix}$

d $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 7 \\ -6 \\ -5 \end{pmatrix} + \mu\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$

e $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + \mu\begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix}$

f $\mathbf{r} = \begin{pmatrix} 0 \\ 7 \\ -2 \end{pmatrix} + \lambda\begin{pmatrix} 6 \\ -4 \\ 8 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -12 \\ -1 \\ 11 \end{pmatrix} + \mu\begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}$