1 Using the substitution $u^2 = x + 3$, show that

$$\int_0^1 x\sqrt{x+3}\ \mathrm{d}x = k(3\sqrt{3} - 4),$$

where k is a rational number to be found. **(7)**

2 **a** Find the values of the constants p, q and r such that

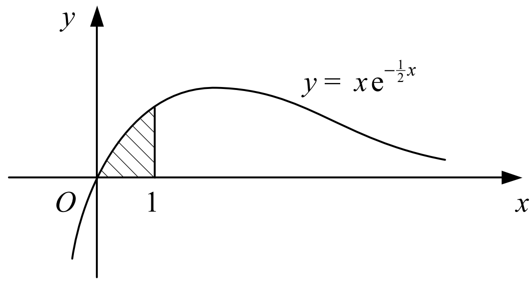
$$\sin^4 x \equiv p + q\cos 2x + r\cos 4x.$$ **(3)**

b Hence, evaluate

$$\int_0^{\frac{\pi}{2}} \sin^4 x\ \mathrm{d}x,$$

giving your answer in terms of π. **(4)**

3



The shaded region in the diagram, bounded by the curve $y = xe^{-\frac{1}{2}x}$, the x-axis and the line $x = 1$, is rotated through four right angles about the x-axis.

Show that the volume of the solid formed is $\pi(2 - 5e^{-1})$. **(7)**

4 **a** Find

$$\int 6x\cos 3x\ \mathrm{d}x.$$ **(4)**

b Use the substitution $x = 2\sin u$ to find

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\ \mathrm{d}x,$$

giving your answer in terms of π. **(5)**

5

$$\mathrm{f}(x) \equiv \frac{6-2x^2}{(x+1)^2(x+3)}.$$

a Find the values of the constants A, B and C such that

$$\mathrm{f}(x) \equiv \frac{A}{(x+1)^2} + \frac{B}{x+1} + \frac{C}{x+3}.$$ **(5)**

The curve $y = \mathrm{f}(x)$ crosses the y-axis at the point P.

b Show that the tangent to the curve at P has the equation

$$14x + 3y = 6.$$ **(4)**

c Evaluate

$$\int_0^1 \mathrm{f}(x)\ \mathrm{d}x,$$

giving your answer in the form $a + b\ln 2 + c\ln 3$ where a, b and c are integers. **(5)**

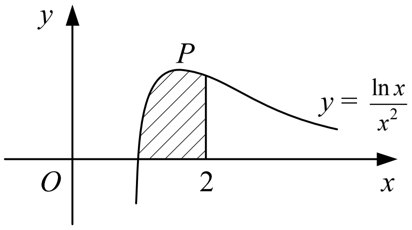
6 **a** Use the identity for $\sin(A + B)$ to prove that

$$\sin 2A \equiv 2 \sin A \cos A. \qquad \textbf{(2)}$$

b Evaluate

$$\int_0^{\frac{\pi}{4}} \sin 4t \cos 2t \;\mathrm{d}t. \qquad \textbf{(5)}$$

7



The shaded region in the diagram is bounded by the curve with equation $y = \frac{\ln x}{x^2}$, the x-axis and the line $x = 2$.

Find the exact area of the shaded region. **(6)**

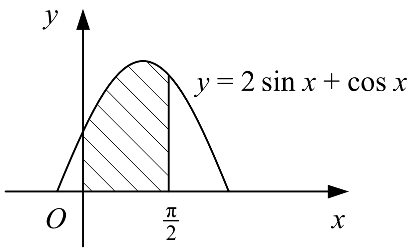
8 **a** Show that, using the substitution $x = \mathrm{e}^u$,

$$\int \frac{2 + \ln x}{x^2} \;\mathrm{d}x = \int (2 + u)\mathrm{e}^{-u} \;\mathrm{d}u. \qquad \textbf{(3)}$$

b Hence, or otherwise, evaluate

$$\int_1^{\mathrm{e}} \frac{2 + \ln x}{x^2} \;\mathrm{d}x. \qquad \textbf{(5)}$$

9



The diagram shows part of the curve with equation $y = 2 \sin x + \cos x$.

The shaded region is bounded by the curve in the interval $0 \le x < \frac{\pi}{2}$, the positive coordinate axes and the line $x = \frac{\pi}{2}$.

a Find the area of the shaded region. **(4)**

b Show that the volume of the solid formed when the shaded region is rotated through 2π radians about the x-axis is $\frac{1}{4}\pi(5\pi + 8)$. **(6)**

10

$$\mathrm{f}(x) \equiv \frac{5x + 1}{(1 - x)(1 + 2x)}.$$

a Express $\mathrm{f}(x)$ in partial fractions. **(3)**

b Find $\int_0^{\frac{1}{2}} \mathrm{f}(x) \;\mathrm{d}x$, giving your answer in the form $k \ln 2$. **(4)**

c Find the series expansion of $\mathrm{f}(x)$ in ascending powers of x up to and including the term in x^3, for $|x| < \frac{1}{2}$. **(5)**