1 Using an appropriate method, integrate with respect to x

a $(2x-3)^4$ **b** $\text{cosec}^2 \tfrac{1}{2}x$ **c** $2e^{4x-1}$ **d** $\dfrac{2(x-1)}{x(x+1)}$

e $\dfrac{3}{\cos^2 2x}$ **f** $x(x^2+3)^3$ **g** $\sec^4 x \tan x$ **h** $\sqrt{7+2x}$

i xe^{3x} **j** $\dfrac{x+2}{x^2-2x-3}$ **k** $\dfrac{1}{4(x+1)^3}$ **l** $\tan^2 3x$

m $4\cos^2(2x+1)$ **n** $\dfrac{3x}{1-x^2}$ **o** $x\sin 2x$ **p** $\dfrac{x+4}{x+2}$

2 Evaluate

a $\displaystyle\int_1^2 6e^{2x-3}\,dx$ **b** $\displaystyle\int_0^{\frac{\pi}{3}} \tan x\,dx$ **c** $\displaystyle\int_{-2}^2 \frac{2}{x-3}\,dx$

d $\displaystyle\int_2^3 \frac{6+x}{4+3x-x^2}\,dx$ **e** $\displaystyle\int_1^2 (1-2x)^3\,dx$ **f** $\displaystyle\int_0^{\frac{\pi}{3}} \sin^2 x \sin 2x\,dx$

3 Using the given substitution, evaluate

a $\displaystyle\int_0^{\frac{3}{2}} \frac{1}{\sqrt{9-x^2}}\,dx$ $x = 3\sin u$ **b** $\displaystyle\int_0^1 x(1-3x)^3\,dx$ $u = 1-3x$

c $\displaystyle\int_2^{2\sqrt{3}} \frac{1}{4+x^2}\,dx$ $x = 2\tan u$ **d** $\displaystyle\int_{-1}^0 x^2\sqrt{x+1}\,dx$ $u^2 = x+1$

4 Integrate with respect to x

a $\dfrac{2}{5-3x}$ **b** $(x+1)e^{x^2+2x}$ **c** $\dfrac{1-x}{2x+1}$ **d** $\sin 3x \cos 2x$

e $3x(x-1)^4$ **f** $\dfrac{6x-5}{(x-1)(2x-1)^2}$ **g** $\dfrac{5}{\sqrt[3]{2x-1}}$ **h** $\dfrac{\cos x}{2+3\sin x}$

i $\dfrac{x^2}{\sqrt{x^3-1}}$ **j** $(2-\cot x)^2$ **k** $\dfrac{x+5}{(x+1)(x-3)^2}$ **l** x^2e^{-x}

5 Evaluate

a $\displaystyle\int_2^4 \frac{1}{3x-4}\,dx$ **b** $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \text{cosec}^2 x \cot^2 x\,dx$ **c** $\displaystyle\int_0^1 \frac{7-x^2}{(2-x)^2(3-x)}\,dx$

d $\displaystyle\int_0^{\frac{\pi}{2}} x\cos \tfrac{1}{2}x\,dx$ **e** $\displaystyle\int_1^5 \frac{1}{\sqrt{4x+5}}\,dx$ **f** $\displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} 2\cos x \cos 3x\,dx$

g $\displaystyle\int_0^2 x\sqrt{2x^2+1}\,dx$ **h** $\displaystyle\int_1^2 \frac{1}{x(2x-1)^2}\,dx$ **i** $\displaystyle\int_0^1 (x-2)(x+1)^3\,dx$

6 Find the exact area of the region enclosed by the given curve, the x-axis and the given ordinates.

a $y = \dfrac{x}{(x^2+2)^3}$, $x = 1$, $x = 2$ **b** $y = \ln x$, $x = 2$, $x = 4$

7 The region enclosed by the given curve, the x-axis and the given ordinates is rotated through four right angles about the x-axis. Find the exact volume of the solid formed in each case.

a $y = \sqrt{\dfrac{x+3}{x+2}}$, $x = 1$, $x = 4$ **b** $y = x^{\frac{1}{2}}e^{2-x}$, $x = 1$, $x = 2$

8 **a** Evaluate $\int_0^{\frac{\pi}{3}} \sin x \sec^2 x \, dx$.

b Using the substitution $u = \cos\theta$, or otherwise, show that

$$\int_0^{\frac{\pi}{4}} \frac{\sin\theta}{\cos^4\theta} \, d\theta = a + b\sqrt{2},$$

where a and b are rational.

9 **a** Express $\frac{1}{x^2 - 3x + 2}$ in partial fractions.

b Show that

$$\int_3^4 \frac{1}{x^2 - 3x + 2} \, dx = \ln \frac{a}{b},$$

where a and b are integers to be found.

10 A student is trying to find the value of the integral

$$\int_0^{\frac{\pi}{6}} x \sin x \, dx,$$

and uses the approximation $\sin x \approx x - \frac{1}{6}x^3$ before integrating.

a Find to 3 significant figures the value she obtains.

b Use integration by parts to find the exact value of the integral.

c Find, to 2 significant figures, the percentage error in the value obtained in part **a**.

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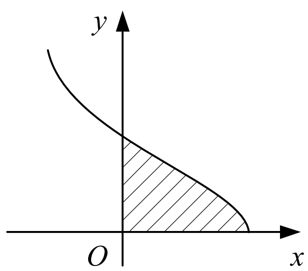

The diagram shows the curve with parametric equations

$$x = \cos 2t, \quad y = \tan t, \quad 0 \le t < \frac{\pi}{2}.$$

a Write down expressions in terms of $\cos 2A$ for

i $\sin^2 A$,
ii $\cos^2 A$,

and hence find a cartesian equation for the curve in the form $y^2 = f(x)$.

The shaded region bounded by the curve and the coordinate axes is rotated through four right angles about the x-axis.

b Show that the volume of the solid formed is $\pi(2 \ln 2 - 1)$.

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$$f(x) \equiv \frac{x + 16}{3x^3 + 11x^2 + 8x - 4}$$

a Factorise completely $3x^3 + 11x^2 + 8x - 4$.

b Express $f(x)$ in partial fractions.

c Show that $\int_{-1}^{0} f(x) \, dx = -(1 + 3 \ln 2)$.