C4 DIFFERENTIATION — Worksheet D

1 Differentiate with respect to x

a $4y$ **b** y^3 **c** $\sin 2y$ **d** $3e^{y^2}$

2 Find $\frac{dy}{dx}$ in terms of x and y in each case.

a $x^2 + y^2 = 2$ **b** $2x - y + y^2 = 0$ **c** $y^4 = x^2 - 6x + 2$

d $x^2 + y^2 + 3x - 4y = 9$ **e** $x^2 - 2y^2 + x + 3y - 4 = 0$ **f** $\sin x + \cos y = 0$

g $2e^{3x} + e^{-2y} + 7 = 0$ **h** $\tan x + \operatorname{cosec} 2y = 1$ **i** $\ln (x - 2) = \ln (2y + 1)$

3 Differentiate with respect to x

a xy **b** x^2y^3 **c** $\sin x \tan y$ **d** $(x - 2y)^3$

4 Find $\frac{dy}{dx}$ in terms of x and y in each case.

a $x^2y = 2$ **b** $x^2 + 3xy - y^2 = 0$ **c** $4x^2 - 2xy + 3y^2 = 8$

d $\cos 2x \sec 3y + 1 = 0$ **e** $y = (x + y)^2$ **f** $xe^y - y = 5$

g $2xy^2 - x^3y = 0$ **h** $y^2 + x \ln y = 3$ **i** $x \sin y + x^2 \cos y = 1$

5 Find an equation for the tangent to each curve at the given point on the curve.

a $x^2 + y^2 - 3y - 2 = 0$, $(2, 1)$ **b** $2x^2 - xy + y^2 = 28$, $(3, 5)$

c $4 \sin y - \sec x = 0$, $(\frac{\pi}{3}, \frac{\pi}{6})$ **d** $2 \tan x \cos y = 1$, $(\frac{\pi}{4}, \frac{\pi}{3})$

6 A curve has the equation $x^2 + 2y^2 - x + 4y = 6$.

a Show that $\frac{dy}{dx} = \frac{1-2x}{4(y+1)}$.

b Find an equation for the normal to the curve at the point $(1, -3)$.

7 A curve has the equation $x^2 + 4xy - 3y^2 = 36$.

a Find an equation for the tangent to the curve at the point P (4, 2).

Given that the tangent to the curve at the point Q on the curve is parallel to the tangent at P,

b find the coordinates of Q.

8 A curve has the equation $y = a^x$, where a is a positive constant.

By first taking logarithms, find an expression for $\frac{dy}{dx}$ in terms of a and x.

9 Differentiate with respect to x

a 3^x **b** 6^{2x} **c** 5^{1-x} **d** 2^{x^3}

10 The curve with equation $y = 3x - 2^x$ has a stationary point at A.

a Find the x-coordinate of A.

b Show that at A, $\frac{d^2y}{dx^2} = -3 \ln 2$, and hence, deduce the nature of the stationary point.

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