**1** Find the values of the constants $A$ and $B$ in each identity.

**a** $x - 8 \equiv A(x - 2) + B(x + 4)$ **b** $6x + 7 \equiv A(2x - 1) + B(x + 2)$

**2** Find the values of the constants $A$ and $B$ in each identity.

**a** $\dfrac{2}{(x+1)(x+3)} \equiv \dfrac{A}{x+1} + \dfrac{B}{x+3}$ **b** $\dfrac{x-3}{x(x-1)} \equiv \dfrac{A}{x} + \dfrac{B}{x-1}$

**c** $\dfrac{x+1}{(x-3)(x-5)} \equiv \dfrac{A}{x-3} + \dfrac{B}{x-5}$ **d** $\dfrac{x+10}{(1+x)(2-x)} \equiv \dfrac{A}{1+x} + \dfrac{B}{2-x}$

**e** $\dfrac{4x-1}{x^2+x-2} \equiv \dfrac{A}{x+2} + \dfrac{B}{x-1}$ **f** $\dfrac{x-9}{x^2-4x+3} \equiv \dfrac{A}{x-1} + \dfrac{B}{x-3}$

**3** Express in partial fractions

**a** $\dfrac{8}{(x-1)(x+3)}$ **b** $\dfrac{x-1}{(x+2)(x+3)}$ **c** $\dfrac{10x}{(x+4)(x-1)}$

**d** $\dfrac{5x+7}{x^2+x}$ **e** $\dfrac{x+2}{x^2-5x+4}$ **f** $\dfrac{4x+6}{x^2-9}$

**g** $\dfrac{3x+2}{x^2-2x-24}$ **h** $\dfrac{38-x}{12-x-x^2}$ **i** $\dfrac{4x-5}{(2x+1)(x-3)}$

**j** $\dfrac{1-3x}{(3x+4)(2x+1)}$ **k** $\dfrac{x+1}{x-3x^2}$ **l** $\dfrac{5}{2x^2+3x-2}$

**m** $\dfrac{2(x+5)}{8x^2+10x-3}$ **n** $\dfrac{3x-7}{x^2-2x-3}$ **o** $\dfrac{1-3x}{1-x-2x^2}$

**4** Find the values of the constants $A$, $B$ and $C$ in each identity.

**a** $3x^2 + 17x - 32 \equiv A(x - 1)(x + 3) + B(x - 1)(x - 4) + C(x + 3)(x - 4)$

**b** $14x + 2 \equiv A(x + 1)(x - 2) + B(x + 1)(3x - 1) + C(x - 2)(3x - 1)$

**c** $x^2 + x + 12 \equiv A(x + 1)^2 + B(x + 1)(x + 5) + C(x + 5)$

**d** $4(5x^2 + 4) \equiv A(2x + 1)^2 + B(2x + 1)(x - 3) + C(x - 3)$

**5** Find the values of the constants $A$, $B$ and $C$ in each identity.

**a** $\dfrac{8x+14}{(x-2)(x+1)(x+3)} \equiv \dfrac{A}{x-2} + \dfrac{B}{x+1} + \dfrac{C}{x+3}$

**b** $\dfrac{2x^2-6x+20}{(x+1)(x+2)(x-6)} \equiv \dfrac{A}{x+1} + \dfrac{B}{x+2} + \dfrac{C}{x-6}$

**c** $\dfrac{9x-14}{(x+4)(x-1)^2} \equiv \dfrac{A}{x+4} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$

**d** $\dfrac{3x^2-7x-4}{(x-3)(x-2)^2} \equiv \dfrac{A}{x-3} + \dfrac{B}{x-2} + \dfrac{C}{(x-2)^2}$

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**6** Express in partial fractions

**a** $\dfrac{2x^2+4}{x(x-1)(x-4)}$ **b** $\dfrac{9}{(x-2)(x+1)^2}$ **c** $\dfrac{x^2+11x-21}{(2x+1)(x-2)(x-3)}$

**d** $\dfrac{10x+9}{(x-4)(x+3)^2}$ **e** $\dfrac{x^2+4x+5}{(x+1)(x+2)^2}$ **f** $\dfrac{16-2x}{(x-3)(x^2-4)}$

**g** $\dfrac{2-9x}{(x-3)(2x-1)^2}$ **h** $\dfrac{3+24x-4x^2}{(x+1)(x-4)^2}$ **i** $\dfrac{9x^2-2x-12}{x^3+x^2-6x}$

**j** $\dfrac{5x^2+3x-20}{x^3+4x^2}$ **k** $\dfrac{13-3x^2}{(2x+3)(x-1)^2}$ **l** $\dfrac{26-x-x^2}{(x-1)(x+3)(x+5)}$

**7** Given that

$$\mathrm{f}(x) = 3x^3 + 11x^2 + 8x - 4,$$

**a** fully factorise $\mathrm{f}(x)$,

**b** express $\dfrac{x+16}{\mathrm{f}(x)}$ in partial fractions.

**8** The function f is defined by

$$\mathrm{f}(x) = \frac{4}{x^2-1}.$$

**a** Express $\mathrm{f}(x)$ in partial fractions.

The function g is defined by

$$\mathrm{g}(x) = \frac{2+5x-x^2}{(x-4)(x-2)(x-1)}.$$

**b** Express $\mathrm{g}(x)$ in partial fractions.

**c** Hence, or otherwise, solve the equation $\mathrm{f}(x) = \mathrm{g}(x)$.

**9**

$$\mathrm{f}(x) = \frac{x+5}{(x-1)(2x+1)}.$$

**a** Express $\mathrm{f}(x)$ in partial fractions.

**b** Find the $x$-coordinates of the stationary points of the curve $y = \mathrm{f}(x)$, giving your answers in the form $a + b\sqrt{3}$, where $a$ and $b$ are integers.

**10** The function f is defined by

$$\mathrm{f}(x) = \frac{x(4x+5)}{(x-1)(x+2)^2}.$$

**a** Find the values of the constants $A$, $B$ and $C$ such that

$$\mathrm{f}(x) = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}.$$

**b** Show that the tangent to the curve $y = \mathrm{f}(x)$ at the point where $x = -1$ has the equation

$$3x - 4y + 5 = 0.$$

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