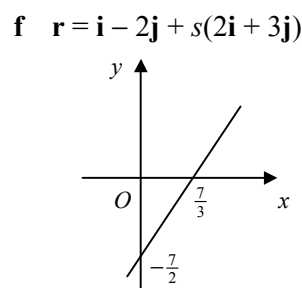
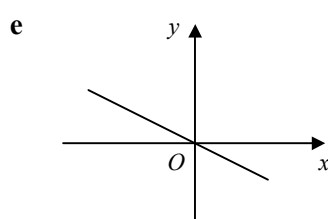
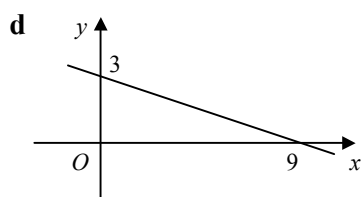
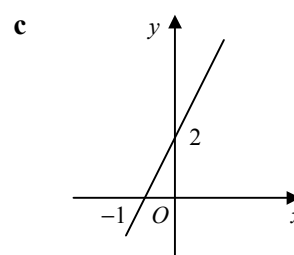
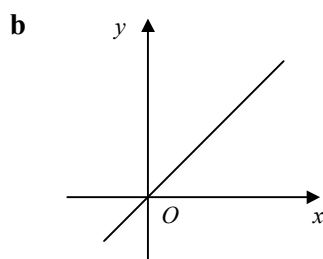
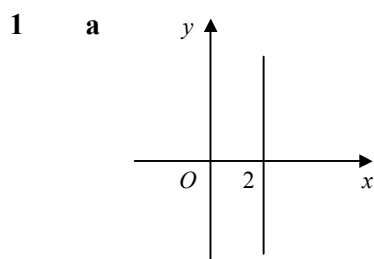


**Note:** For this worksheet especially, there may be alternative correct answers



**2 a**  $\mathbf{r} = -\mathbf{i} + \mathbf{j} + s(3\mathbf{i} - 2\mathbf{j})$

**b**  $\mathbf{r} = 4\mathbf{j} + s\mathbf{i}$

**c**  $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + s(\mathbf{i} + 5\mathbf{j})$

**3 a**  $\text{dir}^n = \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$   
 $\therefore \mathbf{r} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

**b**  $\text{dir}^n = \begin{pmatrix} -1 \\ 1 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$   
 $\therefore \mathbf{r} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} + s \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

**c**  $\text{dir}^n = \begin{pmatrix} -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$   
 $\therefore \mathbf{r} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} + s \begin{pmatrix} -4 \\ 5 \end{pmatrix}$

**4 a**  $-1 + 2\lambda = 5 \therefore \lambda = 3$   
 $3 + c\lambda = 3 + 3c = 0 \therefore c = -1$

**b**  $c\mathbf{i} + 2\mathbf{j} = k(6\mathbf{i} + 3\mathbf{j})$   
 $\therefore k = \frac{2}{3}$   
 $\therefore c = 4$

**5 a**  $\mathbf{r} = -\mathbf{i} + s\mathbf{j}$

**b**  $\mathbf{r} = s(\mathbf{i} + 2\mathbf{j})$

**c**  $\mathbf{r} = \mathbf{j} + s(\mathbf{i} + 3\mathbf{j})$

**d**  $\mathbf{r} = -2\mathbf{j} + s(4\mathbf{i} + 3\mathbf{j})$

**e**  $\mathbf{r} = 5\mathbf{j} + s(2\mathbf{i} - \mathbf{j})$

**f**  $y = \frac{1}{4}x + 2$   
 $\therefore \mathbf{r} = 2\mathbf{j} + s(4\mathbf{i} + \mathbf{j})$

**6 a**  $x = 2 + 3\lambda, y = 1 + 2\lambda$

**b**  $\lambda = \frac{x-2}{3} = \frac{y-1}{2}$   
 $2(x-2) = 3(y-1)$   
 $2x - 4 = 3y - 3$   
 $2x - 3y - 1 = 0$

- 7    **a**  $x = 3 + \lambda, y = 2\lambda$   
 $\lambda = x - 3 = \frac{y}{2}$   
 $2(x - 3) = y$   
 $2x - y - 6 = 0$
- b**  $x = 1 + 3\lambda, y = 4 + \lambda$   
 $\lambda = \frac{x-1}{3} = y - 4$   
 $x - 1 = 3(y - 4)$   
 $x - 3y + 11 = 0$
- c**  $x = 4\lambda, y = 2 - \lambda$   
 $\lambda = \frac{x}{4} = 2 - y$   
 $x = 4(2 - y)$   
 $x + 4y - 8 = 0$
- d**  $x = -2 + 5\lambda, y = 1 + 2\lambda$   
 $\lambda = \frac{x+2}{5} = \frac{y-1}{2}$   
 $2(x+2) = 5(y-1)$   
 $2x - 5y + 9 = 0$
- e**  $x = 2 - 3\lambda, y = -3 + 4\lambda$   
 $\lambda = \frac{x-2}{-3} = \frac{y+3}{4}$   
 $4(x-2) = -3(y+3)$   
 $4x + 3y + 1 = 0$
- f**  $x = \lambda + 3, y = -2\lambda - 1$   
 $\lambda = x - 3 = \frac{y+1}{-2}$   
 $-2(x-3) = y+1$   
 $2x + y - 5 = 0$
- 8    **a**  $\begin{pmatrix} 3 \\ -1 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} -6 \\ 2 \end{pmatrix}$   
 $\therefore$  parallel  
 $(1, 2)$  lies on first line  
when  $x = 1$  on second line  
 $-2 - 6t = 1 \Rightarrow t = -\frac{1}{2}$   
 $\Rightarrow y = 3 + 2(-\frac{1}{2}) = 2$   
parallel and common point  
 $\therefore$  identical
- b**  $\begin{pmatrix} 1 \\ 4 \end{pmatrix} \neq k \begin{pmatrix} 4 \\ 1 \end{pmatrix}$   
 $\therefore$  not parallel
- c**  $\begin{pmatrix} 2 \\ 4 \end{pmatrix} = \frac{2}{3} \begin{pmatrix} 3 \\ 6 \end{pmatrix}$   
 $\therefore$  parallel  
 $(2, -5)$  lies on first line  
when  $x = 2$  on second line  
 $-1 + 3t = 2 \Rightarrow t = 1$   
 $\Rightarrow y = 1 + 6(1) = 7$   
 $\therefore (2, -5)$  not on second line  
 $\therefore$  parallel but not identical
- 9    **a**  $1 + \lambda = 2 + 3\mu$  (1)  
 $2 = 1 + \mu$  (2)  
(2)  $\Rightarrow \mu = 1$   
 $\therefore 5\mathbf{i} + 2\mathbf{j}$
- b**  $4 - \lambda = 5 + 2\mu$  (1)  
 $1 + \lambda = -2 - 3\mu$  (2)  
(1) + (2)  $\Rightarrow 5 = 3 - \mu$   
 $\mu = -2$   
 $\therefore \mathbf{i} + 4\mathbf{j}$
- c**  $2\lambda = 2 - \mu$  (1)  
 $1 - \lambda = 10 + 3\mu$  (2)  
(1) + 2×(2)  $\Rightarrow 2 = 22 + 5\mu$   
 $\mu = -4$   
 $\therefore 6\mathbf{i} - 2\mathbf{j}$
- d**  $-1 - 4\lambda = 2 - \mu$  (1)  
 $5 + 6\lambda = -2 + 2\mu$  (2)  
 $2 \times (1) + (2) \Rightarrow 3 - 2\lambda = 2$   
 $\lambda = \frac{1}{2}$   
 $\therefore -3\mathbf{i} + 8\mathbf{j}$
- e**  $-2 - 3\lambda = -3 + 5\mu$  (1)  
 $11 + 4\lambda = -7 + 3\mu$  (2)  
 $4 \times (1) + 3 \times (2)$   
 $\Rightarrow 25 = -33 + 29\mu$   
 $\mu = 2$   
 $\therefore 7\mathbf{i} - \mathbf{j}$
- f**  $1 + 3\lambda = 3 + \mu$  (1)  
 $2 + 2\lambda = 5 + 4\mu$  (2)  
 $2 \times (1) - 3 \times (2)$   
 $\Rightarrow -4 = -9 - 10\mu$   
 $\mu = -\frac{1}{2}$   
 $\therefore \frac{5}{2}\mathbf{i} + 3\mathbf{j}$
- 10    **a**  $\mathbf{r} = 4\mathbf{i} + \mathbf{k} + s(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$   
**b**  $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + s\mathbf{k}$   
**c**  $\mathbf{r} = -\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} + s(2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$
- 11    **a**  $\overline{AB} = (6\mathbf{i} - 3\mathbf{j} + \mathbf{k}) - (5\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = \mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$   
**b**  $\mathbf{r} = (5\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + s(\mathbf{i} - 4\mathbf{j} + 3\mathbf{k})$   
**c**  $5 + s = 3 \Rightarrow s = -2$   
when  $s = -2, \mathbf{r} = (5\mathbf{i} + \mathbf{j} - 2\mathbf{k}) - 2(\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}) = 3\mathbf{i} + 9\mathbf{j} - 8\mathbf{k}$   
 $\therefore l$  passes through  $(3, 9, -8)$

- 12** a direction =  $(5\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}) - (\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$   
 $= 4\mathbf{i} + \mathbf{j} + 2\mathbf{k}$   
 $\therefore \mathbf{r} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + s(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$
- b direction =  $(\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}) - (3\mathbf{i} - 2\mathbf{k})$   
 $= -2\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$   
 $\therefore \mathbf{r} = 3\mathbf{i} - 2\mathbf{k} + s(-2\mathbf{i} + 5\mathbf{j} + 4\mathbf{k})$
- c  $\mathbf{r} = s(6\mathbf{i} - \mathbf{j} + 2\mathbf{k})$
- d direction =  $(4\mathbf{i} - 7\mathbf{j} + \mathbf{k}) - (-\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$   
 $= 5\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$   
 $\therefore \mathbf{r} = -\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + s(5\mathbf{i} - 5\mathbf{j} - 2\mathbf{k})$
- 13** a  $3 + 2\lambda = 9 \therefore \lambda = 3$   
 $-5 + a\lambda = -5 + 3a = -2 \therefore a = 1$   
 $1 + b\lambda = 1 + 3b = -8 \therefore b = -3$
- b  $2\mathbf{i} + a\mathbf{j} + b\mathbf{k} = k(8\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$   
 $\therefore k = \frac{1}{4}$   
 $\therefore a = -1, b = \frac{1}{2}$
- 14** a  $x = 2 + 3\lambda,$   
 $y = 3 + 5\lambda,$   
 $z = 2\lambda,$   
 $(\lambda =) \frac{x-2}{3} = \frac{y-3}{5} = \frac{z}{2}$
- b  $x = 4 + \lambda,$   
 $y = -1 + 6\lambda,$   
 $z = 3 + 3\lambda,$   
 $(\lambda =) x - 4 = \frac{y+1}{6} = \frac{z-3}{3}$
- c  $x = -1 + 4\lambda,$   
 $y = 5 - 2\lambda,$   
 $z = -2 - \lambda,$   
 $(\lambda =) \frac{x+1}{4} = \frac{y-5}{-2} = \frac{z+2}{-1}$
- 15** a  $s = \frac{x-1}{3} = \frac{y+4}{2} = z - 5$   
 $x = 1 + 3s,$   
 $y = -4 + 2s,$   
 $z = 5 + s,$   
 $\mathbf{r} = \mathbf{i} - 4\mathbf{j} + 5\mathbf{k} + s(3\mathbf{i} + 2\mathbf{j} + \mathbf{k})$
- b  $s = \frac{x}{4} = \frac{y-1}{-2} = \frac{z+7}{3}$   
 $x = 4s,$   
 $y = 1 - 2s,$   
 $z = -7 + 3s,$   
 $\mathbf{r} = \mathbf{j} - 7\mathbf{k} + s(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$
- c  $s = \frac{x+5}{-4} = y + 3 = z$   
 $x = -5 - 4s,$   
 $y = -3 + s,$   
 $z = s,$   
 $\mathbf{r} = -5\mathbf{i} - 3\mathbf{j} + s(-4\mathbf{i} + \mathbf{j} + \mathbf{k})$
- 16**  $4 + s = 7 - 3t$  (1)  
 $-2s = 2 + 2t$  (2)  
 $3 + 2s = -5 + t$  (3)  
 $(2) + (3) \Rightarrow 3 = -3 + 3t$   
 $t = 2, s = -3$   
 check (1)  $4 + (-3) = 7 - 3(2)$   
 true  $\therefore$  intersect  
 point of intersection:  $(1, 6, -3)$
- 17**  $2 + \lambda = 1 + \mu$  (1)  
 $-1 + \lambda = 4 - 2\mu$  (2)  
 $4 + 3\lambda = 3 + \mu$  (3)  
 $(1) - (2) \Rightarrow 3 = -3 + 3\mu$   
 $\mu = 2, \lambda = 1$   
 check (3)  $4 + 3(1) = 3 + (2)$   
 false  $\therefore$  do not intersect  
 $(\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \neq k(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \therefore$  not parallel  
 $\therefore$  skew

18 a  $3 + 4\lambda = 3 + \mu$  (1)

$$1 + \lambda = 2 \quad (2)$$

$$5 - \lambda = -4 + 2\mu \quad (3)$$

$$(2) \Rightarrow \lambda = 1$$

$$\text{sub. (1)} \quad \mu = 4$$

$$\text{check (3)} \quad 5 - (1) = -4 + 2(4)$$

true  $\therefore$  intersect

$$\text{position vector of intersection: } \begin{pmatrix} 7 \\ 2 \\ 4 \end{pmatrix}$$

c  $8 + \lambda = -2 + 4\mu$  (1)

$$2 + 3\lambda = 2 - 3\mu \quad (2)$$

$$-4 - 2\lambda = 8 - 4\mu \quad (3)$$

$$(1) + (3) \Rightarrow 4 - \lambda = 6$$

$$\lambda = -2, \mu = 2$$

$$\text{check (2)} \quad 2 + 3(-2) = 2 - 3(2)$$

true  $\therefore$  intersect

$$\text{position vector of intersection: } \begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$$

e  $4 + 2\lambda = 3 + 5\mu$  (1)

$$-1 + 5\lambda = -2 - 3\mu \quad (2)$$

$$3 - 3\lambda = 1 - 4\mu \quad (3)$$

$$3 \times (1) + 2 \times (3) \Rightarrow 18 = 11 + 7\mu$$

$$\mu = 1, \lambda = 2$$

$$\text{check (2)} \quad -1 + 5(2) = -2 - 3(1)$$

false  $\therefore$  do not intersect

$$\begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix} \neq k \begin{pmatrix} 5 \\ -3 \\ -4 \end{pmatrix}$$

$\therefore$  skew

b  $\begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} -4 \\ 2 \\ 6 \end{pmatrix}$

$\therefore$  parallel

d  $1 + \lambda = 7 + 2\mu$  (1)

$$5 + 4\lambda = -6 + \mu \quad (2)$$

$$2 - 2\lambda = -5 - 3\mu \quad (3)$$

$$2 \times (1) + (3) \Rightarrow 4 = 9 + \mu$$

$$\mu = -5, \lambda = -4$$

$$\text{check (2)} \quad 5 + 4(-4) = -6 + (-5)$$

true  $\therefore$  intersect

$$\text{position vector of intersection: } \begin{pmatrix} -3 \\ -11 \\ 10 \end{pmatrix}$$

f  $6\lambda = -12 + 5\mu$  (1)

$$7 - 4\lambda = -1 + 2\mu \quad (2)$$

$$-2 + 8\lambda = 11 - 3\mu \quad (3)$$

$$2 \times (2) + (3) \Rightarrow 12 = 9 + \mu$$

$$\mu = 3, \lambda = \frac{1}{2}$$

$$\text{check (1)} \quad 6\left(\frac{1}{2}\right) = -12 + 5(3)$$

true  $\therefore$  intersect

$$\text{position vector of intersection: } \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$$