1 **a** $= \frac{1}{2}\times\frac{1}{5}(2x-3)^5 + c$

$= \frac{1}{10}(2x-3)^5 + c$

b $= -2\cot\frac{1}{2}x + c$

c $= \frac{1}{2}e^{4x-1} + c$

d $\frac{2(x-1)}{x(x+1)} \equiv \frac{A}{x} + \frac{B}{x+1}$, $2(x-1) \equiv A(x+1) + Bx$

$x = 0 \Rightarrow A = -2$, $x = -1 \Rightarrow B = 4$

$\int \frac{2(x-1)}{x(x+1)}\,dx = \int \left(\frac{4}{x+1} - \frac{2}{x}\right)dx$

$= 4\ln|x+1| - 2\ln|x| + c$

e $= \int 3\sec^2 2x\,dx$

$= \frac{3}{2}\tan 2x + c$

f $= \frac{1}{2}\int 2x(x^2+3)^3\,dx$

$= \frac{1}{2}\times\frac{1}{4}(x^2+3)^4 + c$

$= \frac{1}{8}(x^2+3)^4 + c$

g $= \int (\sec x\tan x)\sec^3 x\,dx$

$= \frac{1}{4}\sec^4 x + c$

h $= \frac{1}{2}\times\frac{2}{3}(7+2x)^{\frac{3}{2}} + c$

$= \frac{1}{3}(7+2x)^{\frac{3}{2}} + c$

i $u = x,\ \frac{du}{dx} = 1;\ \frac{dv}{dx} = e^{3x},\ v = \frac{1}{3}e^{3x}$

$\int xe^{3x}\,dx = \frac{1}{3}xe^{3x} - \int \frac{1}{3}e^{3x}\,dx$

$= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c$

$= \frac{1}{9}e^{3x}(3x-1) + c$

j $\frac{x+2}{(x-3)(x+1)} \equiv \frac{A}{x-3} + \frac{B}{x+1}$, $x+2 \equiv A(x+1) + B(x-3)$

$x = 3 \Rightarrow A = \frac{5}{4}$, $x = -1 \Rightarrow B = -\frac{1}{4}$

$\int \frac{x+2}{x^2-2x-3}\,dx = \int \left(\frac{\frac{5}{4}}{x-3} - \frac{\frac{1}{4}}{x+1}\right)dx$

$= \frac{5}{4}\ln|x-3| - \frac{1}{4}\ln|x+1| + c$

k $= \frac{1}{4}\times(-\frac{1}{2})(x+1)^{-2} + c$

$= -\frac{1}{8(x+1)^2} + c$

l $= \int (\sec^2 3x - 1)\,dx$

$= \frac{1}{3}\tan 3x - x + c$

m $= \int [2 + 2\cos(4x+2)]\,dx$

$= 2x + \frac{1}{2}\sin(4x+2) + c$

n $= -\frac{3}{2}\int \frac{-2x}{1-x^2}\,dx$

$= -\frac{3}{2}\ln|1-x^2| + c$

o $u = x,\ \frac{du}{dx} = 1;\ \frac{dv}{dx} = \sin 2x,\ v = -\frac{1}{2}\cos 2x$

$\int x\sin 2x\,dx$

$= -\frac{1}{2}x\cos 2x + \int \frac{1}{2}\cos 2x\,dx$

$= -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + c$

p $= \int \frac{(x+2)+2}{x+2}\,dx$

$= \int \left(1 + \frac{2}{x+2}\right)dx$

$= x + 2\ln|x+2| + c$

2 **a** $\int_1^2 6e^{2x-3}\ dx$

$= [3e^{2x-3}]_1^2$

$= 3(e - e^{-1})$

b $\int_0^{\frac{\pi}{3}} \tan x\ dx = -\int_0^{\frac{\pi}{3}} \frac{-\sin x}{\cos x}\ dx$

$= -[\ln|\cos x|]_0^{\frac{\pi}{3}}$

$= -(\ln \frac{1}{2} - 0)$

$= \ln 2$

c $\int_{-2}^2 \frac{2}{x-3}\ dx$

$= [2\ln|x-3|]_{-2}^2$

$= 0 - 2\ln 5$

$= -2\ln 5$

d $\frac{6+x}{(4-x)(1+x)} \equiv \frac{A}{4-x} + \frac{B}{1+x},\ 6+x \equiv A(1+x)+B(4-x)$

$x = 4 \Rightarrow A = 2,\ x = -1 \Rightarrow B = 1$

$\int_2^3 \frac{6+x}{4+3x-x^2}\ dx = \int_2^3 (\frac{2}{4-x} + \frac{1}{1+x})\ dx$

$= [-2\ln|4-x| + \ln|1+x|]_2^3$

$= (0 + \ln 4) - (-2\ln 2 + \ln 3)$

$= 4\ln 2 - \ln 3$

e $\int_1^2 (1-2x)^3\ dx$

$= [-\frac{1}{2}\times\frac{1}{4}(1-2x)^4]_1^2$

$= -\frac{1}{8}(81 - 1)$

$= -10$

f $\int_0^{\frac{\pi}{3}} \sin^2 x \sin 2x\ dx = \int_0^{\frac{\pi}{3}} 2\sin^3 x\cos x\ dx$

$= [\frac{1}{2}\sin^4 x]_0^{\frac{\pi}{3}}$

$= \frac{1}{2}(\frac{\sqrt{3}}{2})^4 - 0$

$= \frac{9}{32}$

3 **a** $x = 3\sin u\ \therefore\ \frac{dx}{du} = 3\cos u$

$x = 0 \Rightarrow u = 0$

$x = \frac{3}{2} \Rightarrow u = \frac{\pi}{6}$

$\int_0^{\frac{3}{2}} \frac{1}{\sqrt{9-x^2}}\ dx = \int_0^{\frac{\pi}{6}} \frac{1}{3\cos u}\times 3\cos u\ du$

$= \int_0^{\frac{\pi}{6}} du$

$= [u]_0^{\frac{\pi}{6}}$

$= \frac{\pi}{6} - 0$

$= \frac{\pi}{6}$

b $u = 1 - 3x\ \therefore\ x = \frac{1}{3}(1-u),\ \frac{du}{dx} = -3$

$x = 0 \Rightarrow u = 1$

$x = 1 \Rightarrow u = -2$

$\int_0^1 x(1-3x)^3\ dx = \int_1^{-2} \frac{1}{3}(1-u)u^3 \times (-\frac{1}{3})\ du$

$= \frac{1}{9}\int_{-2}^1 (u^3 - u^4)\ du$

$= \frac{1}{9}[\frac{1}{4}u^4 - \frac{1}{5}u^5]_{-2}^1$

$= \frac{1}{9}[(\frac{1}{4} - \frac{1}{5}) - (4 + \frac{32}{5})]$

$= -\frac{23}{20}$

c $x = 2\tan u\ \therefore\ \frac{dx}{du} = 2\sec^2 u$

$x = 2 \Rightarrow u = \frac{\pi}{4}$

$x = 2\sqrt{3} \Rightarrow u = \frac{\pi}{3}$

$\int_2^{2\sqrt{3}} \frac{1}{4+x^2}\ dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{4\sec^2 u}\times 2\sec^2 u\ du$

$= \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} du$

$= \frac{1}{2}[u]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$= \frac{1}{2}(\frac{\pi}{3} - \frac{\pi}{4})$

$= \frac{1}{24}\pi$

d $u^2 = x + 1\ \therefore\ x = u^2 - 1,\ \frac{dx}{du} = 2u$

$x = -1 \Rightarrow u = 0$

$x = 0 \Rightarrow u = 1$

$\int_{-1}^0 x^2\sqrt{x+1}\ dx = \int_0^1 (u^2 - 1)^2 u \times 2u\ du$

$= \int_0^1 2u^2(u^4 - 2u^2 + 1)\ du$

$= \int_0^1 (2u^6 - 4u^4 + 2u^2)\ du$

$= [\frac{2}{7}u^7 - \frac{4}{5}u^5 + \frac{2}{3}u^3]_0^1$

$= (\frac{2}{7} - \frac{4}{5} + \frac{2}{3}) - (0)$

$= \frac{16}{105}$

4 **a** $= -\frac{2}{3}\ln|5-3x| + c$

b $= \frac{1}{2}\int (2x+2)\,e^{x^2+2x}\,dx$

$= \frac{1}{2}e^{x^2+2x} + c$

c $= \int \frac{-\frac{1}{2}(2x+1)+\frac{3}{2}}{2x+1}\,dx$

$= \int \left(\frac{\frac{3}{2}}{2x+1} - \frac{1}{2}\right)dx$

$= \frac{3}{4}\ln|2x+1| - \frac{1}{2}x + c$

d $= \frac{1}{2}\int (\sin 5x + \sin x)\,dx$

$= \frac{1}{2}(-\frac{1}{5}\cos 5x - \cos x) + c$

$= -\frac{1}{10}(\cos 5x + 5\cos x) + c$

e $u = 3x,\ \frac{du}{dx} = 3;\ \frac{dv}{dx} = (x-1)^4,\ v = \frac{1}{5}(x-1)^5$

$\int 3x(x-1)^4\,dx$

$= \frac{3}{5}x(x-1)^5 - \int \frac{3}{5}(x-1)^5\,dx$

$= \frac{3}{5}x(x-1)^5 - \frac{1}{10}(x-1)^6 + c$

$= \frac{1}{10}(x-1)^5[6x-(x-1)] + c$

$= \frac{1}{10}(5x+1)(x-1)^5 + c$

f $\frac{6x-5}{(x-1)(2x-1)^2} \equiv \frac{A}{x-1} + \frac{B}{2x-1} + \frac{C}{(2x-1)^2}$

$6x-5 \equiv A(2x-1)^2 + B(x-1)(2x-1) + C(x-1)$

$x = 1 \Rightarrow A = 1,\ x = \frac{1}{2} \Rightarrow C = 4$

coeffs of $x^2 \Rightarrow B = -2$

$\int \frac{6x-5}{(x-1)(2x-1)^2}\,dx$

$= \int \left(\frac{1}{x-1} - \frac{2}{2x-1} + \frac{4}{(2x-1)^2}\right)dx$

$= \ln|x-1| - \ln|2x-1| - 2(2x-1)^{-1} + c$

$= \ln\left|\frac{x-1}{2x-1}\right| - \frac{2}{2x-1} + c$

g $= \int 5(2x-1)^{-\frac{1}{3}}\,dx$

$= \frac{1}{2}\times\frac{15}{2}(2x-1)^{\frac{2}{3}} + c$

$= \frac{15}{4}(2x-1)^{\frac{2}{3}} + c$

h $= \frac{1}{3}\int \frac{3\cos x}{2+3\sin x}\,dx$

$= \frac{1}{3}\ln|2+3\sin x| + c$

i $= \frac{1}{3}\int 3x^2(x^3-1)^{-\frac{1}{2}}\,dx$

$= \frac{1}{3}\times 2(x^3-1)^{\frac{1}{2}} + c$

$= \frac{2}{3}\sqrt{x^3-1} + c$

j $= \int (4 - 4\cot x + \cot^2 x)\,dx$

$= \int \left(4 - 4\frac{\cos x}{\sin x} + \operatorname{cosec}^2 x - 1\right)dx$

$= 3x - 4\ln|\sin x| - \cot x + c$

k $\frac{x+5}{(x+1)(x-3)^2} \equiv \frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

$x+5 \equiv A(x-3)^2 + B(x+1)(x-3) + C(x+1)$

$x = -1 \Rightarrow A = \frac{1}{4},\ x = 3 \Rightarrow C = 2$

coeffs of $x^2 \Rightarrow B = -\frac{1}{4}$

$\int \frac{x+5}{(x+1)(x-3)^2}\,dx$

$= \int \left(\frac{1}{4(x+1)} - \frac{1}{4(x-3)} + \frac{2}{(x-3)^2}\right)dx$

$= \frac{1}{4}\ln|x+1| - \frac{1}{4}\ln|x-3| - 2(x-3)^{-1} + c$

$= \frac{1}{4}\ln\left|\frac{x+1}{x-3}\right| - \frac{2}{x-3} + c$

l $u = x^2,\ \frac{du}{dx} = 2x;\ \frac{dv}{dx} = e^{-x},\ v = -e^{-x}$

$\int x^2e^{-x}\,dx = -x^2e^{-x} + \int 2xe^{-x}\,dx$

$u = 2x,\ \frac{du}{dx} = 2;\ \frac{dv}{dx} = e^{-x},\ v = -e^{-x}$

$\int x^2e^{-x}\,dx = -x^2e^{-x} - 2xe^{-x} + \int 2e^{-x}\,dx$

$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + c$

$= -e^{-x}(x^2 + 2x + 2) + c$

5 **a** $\int_2^4 \frac{1}{3x-4}\, dx$

$= [\frac{1}{3}\ln|3x-4|]_2^4$

$= \frac{1}{3}(\ln 8 - \ln 2)$

$= \frac{2}{3}\ln 2$

b $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \text{cosec}^2 x \cot^2 x\, dx = -\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (-\text{cosec}^2 x)\cot^2 x\, dx$

$= -[\frac{1}{3}\cot^3 x]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$

$= -\frac{1}{3}[1 - (\sqrt{3})^3]$

$= \sqrt{3} - \frac{1}{3}$

c $\frac{7-x^2}{(2-x)^2(3-x)} \equiv \frac{A}{2-x} + \frac{B}{(2-x)^2} + \frac{C}{3-x}$

$7-x^2 \equiv A(2-x)(3-x) + B(3-x) + C(2-x)^2$

$x = 2 \Rightarrow B = 3,\ x = 3 \Rightarrow C = -2$

coeffs of $x^2 \Rightarrow A = 1$

$\int_0^1 \frac{7-x^2}{(2-x)^2(3-x)}\, dx$

$= \int_0^1 (\frac{1}{2-x} + \frac{3}{(2-x)^2} - \frac{2}{3-x})\, dx$

$= [-\ln|2-x| + 3(2-x)^{-1} + 2\ln|3-x|]_0^1$

$= (0 + 3 + 2\ln 2) - (-\ln 2 + \frac{3}{2} + 2\ln 3)$

$= \frac{3}{2} + 3\ln 2 - 2\ln 3$

d $u = x,\ \frac{du}{dx} = 1;\ \frac{dv}{dx} = \cos\frac{1}{2}x,\ v = 2\sin\frac{1}{2}x$

$\int_0^{\frac{\pi}{2}} x\cos\frac{1}{2}x\, dx$

$= [2x\sin\frac{1}{2}x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2\sin\frac{1}{2}x\, dx$

$= [2x\sin\frac{1}{2}x + 4\cos\frac{1}{2}x]_0^{\frac{\pi}{2}}$

$= [\pi(\frac{1}{\sqrt{2}}) - 4(\frac{1}{\sqrt{2}})] - [0 + 4]$

$= \frac{1}{2}\sqrt{2}(\pi - 4) - 4$

e $\int_1^5 \frac{1}{\sqrt{4x+5}}\, dx$

$= [\frac{1}{4}\times 2(4x+5)^{\frac{1}{2}}]_1^5$

$= \frac{1}{2}(5-3)$

$= 1$

f $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} 2\cos x\cos 3x\, dx$

$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} [\cos 4x + \cos(-2x)]\, dx$

$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (\cos 4x + \cos 2x)\, dx$

$= [\frac{1}{4}\sin 4x + \frac{1}{2}\sin 2x]_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$

$= [\frac{1}{4}(\frac{\sqrt{3}}{2}) + \frac{1}{2}(\frac{\sqrt{3}}{2})] - [\frac{1}{4}(-\frac{\sqrt{3}}{2}) + \frac{1}{2}(-\frac{\sqrt{3}}{2})]$

$= \frac{3}{4}\sqrt{3}$

g $\int_0^2 x\sqrt{2x^2+1}\, dx = \frac{1}{4}\int_0^2 4x\sqrt{2x^2+1}\, dx$

$= \frac{1}{4}[\frac{2}{3}(2x^2+1)^{\frac{3}{2}}]_0^2$

$= \frac{1}{6}(27-1)$

$= 4\frac{1}{3}$

h $\frac{1}{x(2x-1)^2} \equiv \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{(2x-1)^2}$

$1 \equiv A(2x-1)^2 + Bx(2x-1) + Cx$

$x = 0 \Rightarrow A = 1,\ x = \frac{1}{2} \Rightarrow C = 2$

coeffs of $x^2 \Rightarrow B = -2$

$\int_1^2 \frac{1}{x(2x-1)^2}\, dx = \int_1^2 (\frac{1}{x} - \frac{2}{2x-1} + \frac{2}{(2x-1)^2})\, dx$

$= [\ln|x| - \ln|2x-1| - (2x-1)^{-1}]_1^2$

$= (\ln 2 - \ln 3 - \frac{1}{3}) - (0 - 0 - 1)$

$= \frac{2}{3} + \ln 2 - \ln 3$

i $u = x - 2,\ \frac{du}{dx} = 1;\ \frac{dv}{dx} = (x+1)^3,\ v = \frac{1}{4}(x+1)^4$

$$\int_0^1 (x-2)(x+1)^3\ dx = [\tfrac{1}{4}(x-2)(x+1)^4]_0^1 - \int_0^1 \tfrac{1}{4}(x+1)^4\ dx$$
$$= [\tfrac{1}{4}(x-2)(x+1)^4 - \tfrac{1}{20}(x+1)^5]_0^1$$
$$= (-4 - \tfrac{8}{5}) - (-\tfrac{1}{2} - \tfrac{1}{20})$$
$$= -5\tfrac{1}{20}$$

6 **a** $= \int_1^2 \frac{x}{(x^2+2)^3}\ dx$

$= \frac{1}{2}\int_1^2 \frac{2x}{(x^2+2)^3}\ dx$

$= \frac{1}{2}[-\frac{1}{2}(x^2+2)^{-2}]_1^2$

$= -\frac{1}{4}(\frac{1}{36} - \frac{1}{9})$

$= \frac{1}{48}$

b $= \int_2^4 \ln x\ dx$

$u = \ln x,\ \frac{du}{dx} = \frac{1}{x};\ \frac{dv}{dx} = 1,\ v = x$

$= [x \ln x]_2^4 - \int_2^4 dx$

$= [x \ln x - x]_2^4$

$= (4 \ln 4 - 4) - (2 \ln 2 - 2)$

$= 6 \ln 2 - 2$

7 **a** $= \pi\int_1^4 (\sqrt{\frac{x+3}{x+2}})^2\ dx$

$= \pi\int_1^4 \frac{x+3}{x+2}\ dx$

$= \pi\int_1^4 \frac{(x+2)+1}{x+2}\ dx$

$= \pi\int_1^4 (1 + \frac{1}{x+2})\ dx$

$= \pi[x + \ln|x+2|]_1^4$

$= \pi[(4 + \ln 6) - (1 + \ln 3)]$

$= \pi(3 + \ln 2)$

b $= \pi\int_1^2 (x^{\frac{1}{2}} e^{2-x})^2\ dx$

$= \pi\int_1^2 xe^{4-2x}\ dx$

$u = x,\ \frac{du}{dx} = 1;\ \frac{dv}{dx} = e^{4-2x},\ v = -\frac{1}{2}e^{4-2x}$

$= \pi\{[-\frac{1}{2}xe^{4-2x}]_1^2 + \int_1^2 \frac{1}{2}e^{4-2x}\ dx\}$

$= \pi[-\frac{1}{2}xe^{4-2x} - \frac{1}{4}e^{4-2x}]_1^2$

$= \pi[(-1 - \frac{1}{4}) - (-\frac{1}{2}e^2 - \frac{1}{4}e^2)]$

$= \frac{1}{4}\pi(3e^2 - 5)$

8 **a** $= \int_0^{\frac{\pi}{3}} \sec x \tan x\ dx$

$= [\sec x]_0^{\frac{\pi}{3}}$

$= 2 - 1$

$= 1$

b $u = \cos\theta\ \therefore\ \frac{du}{d\theta} = -\sin\theta$

$\theta = 0 \Rightarrow u = 1$

$\theta = \frac{\pi}{4} \Rightarrow u = \frac{1}{\sqrt{2}}$

$\int_0^{\frac{\pi}{4}} \frac{\sin\theta}{\cos^4\theta}\ d\theta = \int_1^{\frac{1}{\sqrt{2}}} \frac{1}{u^4} \times (-1)\ du$

$= \int_{\frac{1}{\sqrt{2}}}^1 u^{-4}\ du$

$= [-\frac{1}{3}u^{-3}]_{\frac{1}{\sqrt{2}}}^1$

$= -\frac{1}{3}(1 - 2\sqrt{2})$

$= -\frac{1}{3} + \frac{2}{3}\sqrt{2}\quad [\ a = -\frac{1}{3},\ b = \frac{2}{3}\ ]$

9 **a** $\frac{1}{x^2 - 3x + 2} \equiv \frac{A}{x-1} + \frac{B}{x-2}$

$1 \equiv A(x-2) + B(x-1)$

$x = 1 \Rightarrow 1 = -A \Rightarrow A = -1$

$x = 2 \Rightarrow B = 1$

$\therefore\ \frac{1}{x^2 - 3x + 2} \equiv \frac{1}{x-2} - \frac{1}{x-1}$

b $\int_3^4 \frac{1}{x^2 - 3x + 2}\ dx = \int_3^4 (\frac{1}{x-2} - \frac{1}{x-1})\ dx$

$= [\ln|x-2| - \ln|x-1|]_3^4$

$= (\ln 2 - \ln 3) - (0 - \ln 2)$

$= 2 \ln 2 - \ln 3$

$= \ln \frac{2^2}{3}$

$= \ln \frac{4}{3}\quad [\ a = 4,\ b = 3\ ]$

10 **a** $= \int_0^{\frac{\pi}{6}} x(x - \tfrac{1}{6}x^3)\ \mathrm{d}x$

$= \int_0^{\frac{\pi}{6}} (x^2 - \tfrac{1}{6}x^4)\ \mathrm{d}x$

$= [\tfrac{1}{3}x^3 - \tfrac{1}{30}x^5]_0^{\frac{\pi}{6}}$

$= [\tfrac{1}{3}(\tfrac{\pi}{6})^3 - \tfrac{1}{30}(\tfrac{\pi}{6})^5] - [0]$

$= 0.0465$ (3sf)

b $u = x,\ \frac{\mathrm{d}u}{\mathrm{d}x} = 1;\ \ \frac{\mathrm{d}v}{\mathrm{d}x} = \sin x,\ v = -\cos x$

$\int_0^{\frac{\pi}{6}} x \sin x\ \mathrm{d}x = [-x\cos x]_0^{\frac{\pi}{6}} + \int_0^{\frac{\pi}{6}} \cos x\ \mathrm{d}x$

$= [-x\cos x + \sin x]_0^{\frac{\pi}{6}}$

$= (-\tfrac{\pi}{6} \times \tfrac{\sqrt{3}}{2} + \tfrac{1}{2}) - (0)$

$= \tfrac{1}{12}(6 - \sqrt{3}\,\pi)$

c $= \dfrac{\frac{1}{12}(6-\sqrt{3}\pi) - 0.04654}{\frac{1}{12}(6-\sqrt{3}\pi)} \times 100\%$

$= 0.027\%$ (2sf)

11 **a** **i** $\sin^2 A = \tfrac{1}{2}(1 - \cos 2A)$

ii $\cos^2 A = \tfrac{1}{2}(1 + \cos 2A)$

$y^2 = \tan^2 t = \dfrac{\sin^2 t}{\cos^2 t} = \dfrac{\frac{1}{2}(1-\cos 2t)}{\frac{1}{2}(1+\cos 2t)}$

$\therefore\ y^2 = \dfrac{1-x}{1+x}$

b $y = 0 \Rightarrow \tan t = 0 \Rightarrow t = 0 \Rightarrow x = 1$

volume $= \pi\int_0^1 y^2\ \mathrm{d}x = \pi\int_0^1 \dfrac{1-x}{1+x}\ \mathrm{d}x$

$= \pi\int_0^1 \dfrac{-(1+x)+2}{1+x}\ \mathrm{d}x$

$= \pi\int_0^1 (\dfrac{2}{1+x} - 1)\ \mathrm{d}x$

$= \pi[2\ln|1+x| - x]_0^1$

$= \pi[(2\ln 2 - 1) - (0 - 0)]$

$= \pi(2\ln 2 - 1)$

12 **a** $f(1) = 18,\ f(2) = 80,$

$f(-1) = -4,\ f(-2) = 0$

$\therefore\ (x + 2)$ is a factor

$$
\begin{array}{r|l}
 & 3x^2 + 5x - 2 \\
\hline
x+2 & 3x^3 + 11x^2 + 8x - 4 \\
 & 3x^3 + 6x^2 \\
\hline
 & \quad\ \ 5x^2 + 8x \\
 & \quad\ \ 5x^2 + 10x \\
\hline
 & \qquad\ \ -2x - 4 \\
 & \qquad\ \ -2x - 4 \\
\hline
\end{array}
$$

$\therefore\ 3x^3 + 11x^2 + 8x - 4 = (x + 2)(3x^2 + 5x - 2)$

$= (3x - 1)(x + 2)^2$

b $\dfrac{x+16}{3x^3+11x^2+8x-4} \equiv \dfrac{A}{3x-1} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2}$

$x + 16 \equiv A(x + 2)^2 + B(3x - 1)(x + 2) + C(3x - 1)$

$x = \tfrac{1}{3} \ \Rightarrow\ \tfrac{49}{3} = \tfrac{49}{9}A \ \Rightarrow\ A = 3$

$x = -2 \ \Rightarrow\ 14 = -7C \ \Rightarrow\ C = -2$

coeffs of $x^2 \ \Rightarrow\ 0 = A + 3B \ \Rightarrow\ B = -1$

$\therefore\ f(x) \equiv \dfrac{3}{3x-1} - \dfrac{1}{x+2} - \dfrac{2}{(x+2)^2}$

c $= \int_{-1}^{0} (\dfrac{3}{3x-1} - \dfrac{1}{x+2} - \dfrac{2}{(x+2)^2})\ \mathrm{d}x$

$= [\ln|3x - 1| - \ln|x + 2| + 2(x + 2)^{-1}]_{-1}^{0}$

$= (0 - \ln 2 + 1) - (\ln 4 - 0 + 2)$

$= -1 - \ln 2 - \ln 2^2$

$= -1 - 3\ln 2$

$= -(1 + 3\ln 2)$