

**1 a**  $x = -4 \Rightarrow -12 = -6A \Rightarrow A = 2$   
 $x = 2 \Rightarrow -6 = 6B \Rightarrow B = -1$

**b**  $x = -2 \Rightarrow -5 = -5A \Rightarrow A = 1$   
 $x = \frac{1}{2} \Rightarrow 10 = \frac{5}{2}B \Rightarrow B = 4$

**2 a**  $2 \equiv A(x+3) + B(x+1)$   
 $x = -1 \Rightarrow 2 = 2A \Rightarrow A = 1$   
 $x = -3 \Rightarrow 2 = -2B \Rightarrow B = -1$

**b**  $x - 3 \equiv A(x-1) + Bx$   
 $x = 0 \Rightarrow -3 = -A \Rightarrow A = 3$   
 $x = 1 \Rightarrow \phantom{-3} = \phantom{-A} \Rightarrow B = -2$

**c**  $x + 1 \equiv A(x-5) + B(x-3)$   
 $x = 3 \Rightarrow 4 = -2A \Rightarrow A = -2$   
 $x = 5 \Rightarrow 6 = 2B \Rightarrow B = 3$

**d**  $x + 10 \equiv A(2-x) + B(1+x)$   
 $x = -1 \Rightarrow 9 = 3A \Rightarrow A = 3$   
 $x = 2 \Rightarrow 12 = 3B \Rightarrow B = 4$

**e**  $\frac{4x-1}{(x+2)(x-1)} \equiv \frac{A}{x+2} + \frac{B}{x-1}$   
 $4x - 1 \equiv A(x-1) + B(x+2)$   
 $x = -2 \Rightarrow -9 = -3A \Rightarrow A = 3$   
 $x = 1 \Rightarrow 3 = 3B \Rightarrow B = 1$

**f**  $\frac{x-9}{(x-1)(x-3)} \equiv \frac{A}{x-1} + \frac{B}{x-3}$   
 $x - 9 \equiv A(x-3) + B(x-1)$   
 $x = 1 \Rightarrow -8 = -2A \Rightarrow A = 4$   
 $x = 3 \Rightarrow -6 = 2B \Rightarrow B = -3$

**3 a**  $\frac{8}{(x-1)(x+3)} \equiv \frac{A}{x-1} + \frac{B}{x+3}$   
 $8 \equiv A(x+3) + B(x-1)$   
 $x = 1 \Rightarrow 8 = 4A \Rightarrow A = 2$   
 $x = -3 \Rightarrow 8 = -4B \Rightarrow B = -2$   
 $\therefore \frac{8}{(x-1)(x+3)} \equiv \frac{2}{x-1} - \frac{2}{x+3}$

**b**  $\frac{x-1}{(x+2)(x+3)} \equiv \frac{A}{x+2} + \frac{B}{x+3}$   
 $x - 1 \equiv A(x+3) + B(x+2)$   
 $x = -2 \Rightarrow \phantom{-1} = \phantom{A(x+3)} \Rightarrow A = -3$   
 $x = -3 \Rightarrow -4 = -B \Rightarrow B = 4$   
 $\therefore \frac{x-1}{(x+2)(x+3)} \equiv \frac{4}{x+3} - \frac{3}{x+2}$

**c**  $\frac{10x}{(x+4)(x-1)} \equiv \frac{A}{x+4} + \frac{B}{x-1}$   
 $10x \equiv A(x-1) + B(x+4)$   
 $x = -4 \Rightarrow -40 = -5A \Rightarrow A = 8$   
 $x = 1 \Rightarrow 10 = 5B \Rightarrow B = 2$   
 $\therefore \frac{10x}{(x+4)(x-1)} \equiv \frac{8}{x+4} + \frac{2}{x-1}$

**d**  $\frac{5x+7}{x(x+1)} \equiv \frac{A}{x} + \frac{B}{x+1}$   
 $5x + 7 \equiv A(x+1) + Bx$   
 $x = 0 \Rightarrow 7 = A \Rightarrow A = 7$   
 $x = -1 \Rightarrow 2 = -B \Rightarrow B = -2$   
 $\therefore \frac{5x+7}{x^2+x} \equiv \frac{7}{x} - \frac{2}{x+1}$

**e**  $\frac{x+2}{(x-1)(x-4)} \equiv \frac{A}{x-1} + \frac{B}{x-4}$   
 $x + 2 \equiv A(x-4) + B(x-1)$   
 $x = 1 \Rightarrow 3 = -3A \Rightarrow A = -1$   
 $x = 4 \Rightarrow 6 = 3B \Rightarrow B = 2$   
 $\therefore \frac{x+2}{x^2-5x+4} \equiv \frac{2}{x-4} - \frac{1}{x-1}$

**f**  $\frac{4x+6}{(x+3)(x-3)} \equiv \frac{A}{x+3} + \frac{B}{x-3}$   
 $4x + 6 \equiv A(x-3) + B(x+3)$   
 $x = -3 \Rightarrow -6 = -6A \Rightarrow A = 1$   
 $x = 3 \Rightarrow 18 = 6B \Rightarrow B = 3$   
 $\therefore \frac{4x+6}{x^2-9} \equiv \frac{1}{x+3} + \frac{3}{x-3}$

**g**  $\frac{3x+2}{(x-6)(x+4)} \equiv \frac{A}{x-6} + \frac{B}{x+4}$   
 $3x + 2 \equiv A(x+4) + B(x-6)$   
 $x = 6 \Rightarrow 20 = 10A \Rightarrow A = 2$   
 $x = -4 \Rightarrow -10 = -10B \Rightarrow B = 1$   
 $\therefore \frac{3x+2}{x^2-2x-24} \equiv \frac{2}{x-6} + \frac{1}{x+4}$

**h**  $\frac{38-x}{(4+x)(3-x)} \equiv \frac{A}{4+x} + \frac{B}{3-x}$   
 $38 - x \equiv A(3-x) + B(4+x)$   
 $x = -4 \Rightarrow 42 = 7A \Rightarrow A = 6$   
 $x = 3 \Rightarrow 35 = 7B \Rightarrow B = 5$   
 $\therefore \frac{38-x}{12-x-x^2} \equiv \frac{6}{4+x} + \frac{5}{3-x}$

$$\begin{aligned} \text{i} \quad \frac{4x-5}{(2x+1)(x-3)} &\equiv \frac{A}{2x+1} + \frac{B}{x-3} \\ 4x-5 &\equiv A(x-3) + B(2x+1) \\ x = -\frac{1}{2} &\Rightarrow -7 = -\frac{7}{2}A \Rightarrow A = 2 \\ x = 3 &\Rightarrow 7 = 7B \Rightarrow B = 1 \\ \therefore \frac{4x-5}{(2x+1)(x-3)} &\equiv \frac{2}{2x+1} + \frac{1}{x-3} \end{aligned}$$

$$\begin{aligned} \text{k} \quad \frac{x+1}{x(1-3x)} &\equiv \frac{A}{x} + \frac{B}{1-3x} \\ x+1 &\equiv A(1-3x) + Bx \\ x = 0 &\Rightarrow A = 1 \\ x = \frac{1}{3} &\Rightarrow \frac{4}{3} = \frac{1}{3}B \Rightarrow B = 4 \\ \therefore \frac{x+1}{x-3x^2} &\equiv \frac{1}{x} + \frac{4}{1-3x} \end{aligned}$$

$$\begin{aligned} \text{m} \quad \frac{2x+10}{(4x-1)(2x+3)} &\equiv \frac{A}{4x-1} + \frac{B}{2x+3} \\ 2x+10 &\equiv A(2x+3) + B(4x-1) \\ x = \frac{1}{4} &\Rightarrow \frac{21}{2} = \frac{7}{2}A \Rightarrow A = 3 \\ x = -\frac{3}{2} &\Rightarrow 7 = -7B \Rightarrow B = -1 \\ \therefore \frac{2(x+5)}{8x^2+10x-3} &\equiv \frac{3}{4x-1} - \frac{1}{2x+3} \end{aligned}$$

$$\begin{aligned} \text{o} \quad \frac{1-3x}{(1+x)(1-2x)} &\equiv \frac{A}{1+x} + \frac{B}{1-2x} \\ 1-3x &\equiv A(1-2x) + B(1+x) \\ x = -1 &\Rightarrow 4 = 3A \Rightarrow A = \frac{4}{3} \\ x = \frac{1}{2} &\Rightarrow -\frac{1}{2} = \frac{3}{2}B \Rightarrow B = -\frac{1}{3} \\ \therefore \frac{1-3x}{1-x-2x^2} &\equiv \frac{4}{3(1+x)} - \frac{1}{3(1-2x)} \end{aligned}$$

$$\begin{aligned} 4 \quad \text{a} \quad x = 4 &\Rightarrow 84 = 21A \Rightarrow A = 4 \\ x = -3 &\Rightarrow -56 = 28B \Rightarrow B = -2 \\ x = 1 &\Rightarrow -12 = -12C \Rightarrow C = 1 \\ \text{b} \quad x = \frac{1}{3} &\Rightarrow \frac{20}{3} = -\frac{20}{9}A \Rightarrow A = -3 \\ x = 2 &\Rightarrow 30 = 15B \Rightarrow B = 2 \\ x = -1 &\Rightarrow -12 = 12C \Rightarrow C = -1 \\ \text{c} \quad x = -5 &\Rightarrow 32 = 16A \Rightarrow A = 2 \\ x = -1 &\Rightarrow 12 = 4C \Rightarrow C = 3 \\ \text{coeffs of } x^2 &\Rightarrow 1 = A + B \Rightarrow B = -1 \\ \text{d} \quad x = 3 &\Rightarrow 196 = 49A \Rightarrow A = 4 \\ x = -\frac{1}{2} &\Rightarrow 21 = -\frac{7}{2}C \Rightarrow C = -6 \\ \text{coeffs of } x^2 &\Rightarrow 20 = 4A + 2B \Rightarrow B = 2 \end{aligned}$$

$$\begin{aligned} \text{j} \quad \frac{1-3x}{(3x+4)(2x+1)} &\equiv \frac{A}{3x+4} + \frac{B}{2x+1} \\ 1-3x &\equiv A(2x+1) + B(3x+4) \\ x = -\frac{4}{3} &\Rightarrow 5 = -\frac{5}{3}A \Rightarrow A = -3 \\ x = -\frac{1}{2} &\Rightarrow \frac{5}{2} = \frac{5}{2}B \Rightarrow B = 1 \\ \therefore \frac{1-3x}{(3x+4)(2x+1)} &\equiv \frac{1}{2x+1} - \frac{3}{3x+4} \end{aligned}$$

$$\begin{aligned} \text{l} \quad \frac{5}{(2x-1)(x+2)} &\equiv \frac{A}{2x-1} + \frac{B}{x+2} \\ 5 &\equiv A(x+2) + B(2x-1) \\ x = \frac{1}{2} &\Rightarrow 5 = \frac{5}{2}A \Rightarrow A = 2 \\ x = -2 &\Rightarrow 5 = -5B \Rightarrow B = -1 \\ \therefore \frac{5}{2x^2+3x-2} &\equiv \frac{2}{2x-1} - \frac{1}{x+2} \end{aligned}$$

$$\begin{aligned} \text{n} \quad \frac{3x-7}{(x+1)(x-3)} &\equiv \frac{A}{x+1} + \frac{B}{x-3} \\ 3x-7 &\equiv A(x-3) + B(x+1) \\ x = -1 &\Rightarrow -10 = -4A \Rightarrow A = \frac{5}{2} \\ x = 3 &\Rightarrow 2 = 4B \Rightarrow B = \frac{1}{2} \\ \therefore \frac{3x-7}{x^2-2x-3} &\equiv \frac{5}{2(x+1)} + \frac{1}{2(x-3)} \end{aligned}$$

5 a  $8x + 14 \equiv A(x+1)(x+3) + B(x-2)(x+3) + C(x-2)(x+1)$

$$x = 2 \quad \Rightarrow \quad 30 = 15A \quad \Rightarrow \quad A = 2$$

$$x = -1 \quad \Rightarrow \quad 6 = -6B \quad \Rightarrow \quad B = -1$$

$$x = -3 \quad \Rightarrow \quad -10 = 10C \quad \Rightarrow \quad C = -1$$

b  $2x^2 - 6x + 20 \equiv A(x+2)(x-6) + B(x+1)(x-6) + C(x+1)(x+2)$

$$x = -1 \quad \Rightarrow \quad 28 = -7A \quad \Rightarrow \quad A = -4$$

$$x = -2 \quad \Rightarrow \quad 40 = 8B \quad \Rightarrow \quad B = 5$$

$$x = 6 \quad \Rightarrow \quad 56 = 56C \quad \Rightarrow \quad C = 1$$

c  $9x - 14 \equiv A(x-1)^2 + B(x+4)(x-1) + C(x+4)$

$$x = -4 \quad \Rightarrow \quad -50 = 25A \quad \Rightarrow \quad A = -2$$

$$x = 1 \quad \Rightarrow \quad -5 = 5C \quad \Rightarrow \quad C = -1$$

$$\text{coeffs of } x^2 \Rightarrow 0 = A + B \quad \Rightarrow \quad B = 2$$

d  $3x^2 - 7x - 4 \equiv A(x-2)^2 + B(x-3)(x-2) + C(x-3)$

$$x = 3 \quad \Rightarrow \quad A = 2$$

$$x = 2 \quad \Rightarrow \quad -6 = -C \quad \Rightarrow \quad C = 6$$

$$\text{coeffs of } x^2 \Rightarrow 3 = A + B \quad \Rightarrow \quad B = 1$$

6 a  $\frac{2x^2 + 4}{x(x-1)(x-4)} \equiv \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-4}$

$$2x^2 + 4 \equiv A(x-1)(x-4) + Bx(x-4) + Cx(x-1)$$

$$x = 0 \quad \Rightarrow \quad 4 = 4A \quad \Rightarrow \quad A = 1$$

$$x = 1 \quad \Rightarrow \quad 6 = -3B \quad \Rightarrow \quad B = -2$$

$$x = 4 \quad \Rightarrow \quad 36 = 12C \quad \Rightarrow \quad C = 3 \quad \therefore \frac{2x^2 + 4}{x(x-1)(x-4)} \equiv \frac{1}{x} - \frac{2}{x-1} + \frac{3}{x-4}$$

b  $\frac{9}{(x-2)(x+1)^2} \equiv \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$$9 \equiv A(x+1)^2 + B(x-2)(x+1) + C(x-2)$$

$$x = 2 \quad \Rightarrow \quad 9 = 9A \quad \Rightarrow \quad A = 1$$

$$x = -1 \quad \Rightarrow \quad 9 = -3C \quad \Rightarrow \quad C = -3$$

$$\text{coeffs of } x^2 \Rightarrow 0 = A + B \quad \Rightarrow \quad B = -1 \quad \therefore \frac{9}{(x-2)(x+1)^2} \equiv \frac{1}{x-2} - \frac{1}{x+1} - \frac{3}{(x+1)^2}$$

c  $\frac{x^2 + 11x - 21}{(2x+1)(x-2)(x-3)} \equiv \frac{A}{2x+1} + \frac{B}{x-2} + \frac{C}{x-3}$

$$x^2 + 11x - 21 \equiv A(x-2)(x-3) + B(2x+1)(x-3) + C(2x+1)(x-2)$$

$$x = -\frac{1}{2} \quad \Rightarrow \quad -\frac{105}{4} = \frac{35}{4}A \quad \Rightarrow \quad A = -3$$

$$x = 2 \quad \Rightarrow \quad 5 = -5B \quad \Rightarrow \quad B = -1$$

$$x = 3 \quad \Rightarrow \quad 21 = 7C \quad \Rightarrow \quad C = 3 \quad \therefore \frac{x^2 + 11x - 21}{(2x+1)(x-2)(x-3)} \equiv \frac{3}{x-3} - \frac{3}{2x+1} - \frac{1}{x-2}$$

d  $\frac{10x+9}{(x-4)(x+3)^2} \equiv \frac{A}{x-4} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$

$$10x + 9 \equiv A(x+3)^2 + B(x-4)(x+3) + C(x-4)$$

$$x = 4 \quad \Rightarrow \quad 49 = 49A \quad \Rightarrow \quad A = 1$$

$$x = -3 \quad \Rightarrow \quad -21 = -7C \quad \Rightarrow \quad C = 3$$

$$\text{coeffs of } x^2 \Rightarrow 0 = A + B \quad \Rightarrow \quad B = -1 \quad \therefore \frac{10x+9}{(x-4)(x+3)^2} \equiv \frac{1}{x-4} - \frac{1}{x+3} + \frac{3}{(x+3)^2}$$

- e**  $\frac{x^2+4x+5}{(x+1)(x+2)^2} \equiv \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$   
 $x^2+4x+5 \equiv A(x+2)^2 + B(x+1)(x+2) + C(x+1)$   
 $x = -1 \Rightarrow \quad \quad \quad \Rightarrow A = 2$   
 $x = -2 \Rightarrow 1 = -C \Rightarrow C = -1$   
 coeffs of  $x^2 \Rightarrow 1 = A + B \Rightarrow B = -1 \therefore \frac{x^2+4x+5}{(x+1)(x+2)^2} \equiv \frac{2}{x+1} - \frac{1}{x+2} - \frac{1}{(x+2)^2}$
- f**  $\frac{16-2x}{(x-3)(x+2)(x-2)} \equiv \frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{x-2}$   
 $16-2x \equiv A(x+2)(x-2) + B(x-3)(x-2) + C(x-3)(x+2)$   
 $x = 3 \Rightarrow 10 = 5A \Rightarrow A = 2$   
 $x = -2 \Rightarrow 20 = 20B \Rightarrow B = 1$   
 $x = 2 \Rightarrow 12 = -4C \Rightarrow C = -3 \therefore \frac{16-2x}{(x-3)(x^2-4)} \equiv \frac{2}{x-3} + \frac{1}{x+2} - \frac{3}{x-2}$
- g**  $\frac{2-9x}{(x-3)(2x-1)^2} \equiv \frac{A}{x-3} + \frac{B}{2x-1} + \frac{C}{(2x-1)^2}$   
 $2-9x \equiv A(2x-1)^2 + B(x-3)(2x-1) + C(x-3)$   
 $x = 3 \Rightarrow -25 = 25A \Rightarrow A = -1$   
 $x = \frac{1}{2} \Rightarrow -\frac{5}{2} = -\frac{5}{2}C \Rightarrow C = 1$   
 coeffs of  $x^2 \Rightarrow 0 = 4A + 2B \Rightarrow B = 2 \therefore \frac{2-9x}{(x-3)(2x-1)^2} \equiv \frac{2}{2x-1} + \frac{1}{(2x-1)^2} - \frac{1}{x-3}$
- h**  $\frac{3+24x-4x^2}{(x+1)(x-4)^2} \equiv \frac{A}{x+1} + \frac{B}{x-4} + \frac{C}{(x-4)^2}$   
 $3+24x-4x^2 \equiv A(x-4)^2 + B(x+1)(x-4) + C(x+1)$   
 $x = -1 \Rightarrow -25 = 25A \Rightarrow A = -1$   
 $x = 4 \Rightarrow 35 = 5C \Rightarrow C = 7$   
 coeffs of  $x^2 \Rightarrow -4 = A + B \Rightarrow B = -3 \therefore \frac{3+24x-4x^2}{(x+1)(x-4)^2} \equiv \frac{7}{(x-4)^2} - \frac{3}{x-4} - \frac{1}{x+1}$
- i**  $\frac{9x^2-2x-12}{x(x+3)(x-2)} \equiv \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-2}$   
 $9x^2-2x-12 \equiv A(x+3)(x-2) + Bx(x-2) + Cx(x+3)$   
 $x = 0 \Rightarrow -12 = -6A \Rightarrow A = 2$   
 $x = -3 \Rightarrow 75 = 15B \Rightarrow B = 5$   
 $x = 2 \Rightarrow 20 = 10C \Rightarrow C = 2 \therefore \frac{9x^2-2x-12}{x^3+x^2-6x} \equiv \frac{2}{x} + \frac{5}{x+3} + \frac{2}{x-2}$
- j**  $\frac{5x^2+3x-20}{x^2(x+4)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+4}$   
 $5x^2+3x-20 \equiv Ax(x+4) + B(x+4) + Cx^2$   
 $x = 0 \Rightarrow -20 = 4B \Rightarrow B = -5$   
 $x = -4 \Rightarrow 48 = 16C \Rightarrow C = 3$   
 coeffs of  $x^2 \Rightarrow 5 = A + C \Rightarrow A = 2 \therefore \frac{5x^2+3x-20}{x^3+4x^2} \equiv \frac{2}{x} - \frac{5}{x^2} + \frac{3}{x+4}$
- k**  $\frac{13-3x^2}{(2x+3)(x-1)^2} \equiv \frac{A}{2x+3} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$   
 $13-3x^2 \equiv A(x-1)^2 + B(2x+3)(x-1) + C(2x+3)$   
 $x = -\frac{3}{2} \Rightarrow \frac{25}{4} = \frac{25}{4}A \Rightarrow A = 1$   
 $x = 1 \Rightarrow 10 = 5C \Rightarrow C = 2$   
 coeffs of  $x^2 \Rightarrow -3 = A + 2B \Rightarrow B = -2 \therefore \frac{13-3x^2}{(2x+3)(x-1)^2} \equiv \frac{1}{2x+3} - \frac{2}{x-1} + \frac{2}{(x-1)^2}$

$$1 \quad \frac{26-x-x^2}{(x-1)(x+3)(x+5)} \equiv \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{x+5}$$

$$26-x-x^2 \equiv A(x+3)(x+5) + B(x-1)(x+5) + C(x-1)(x+3)$$

$$x=1 \quad \Rightarrow \quad 24 = 24A \quad \Rightarrow \quad A = 1$$

$$x=-3 \quad \Rightarrow \quad 20 = -8B \quad \Rightarrow \quad B = -\frac{5}{2}$$

$$x=-5 \quad \Rightarrow \quad 6 = 12C \quad \Rightarrow \quad C = \frac{1}{2} \quad \therefore \frac{26-x-x^2}{(x-1)(x+3)(x+5)} \equiv \frac{1}{x-1} - \frac{5}{2(x+3)} + \frac{1}{2(x+5)}$$

$$7 \quad \mathbf{a} \quad f(1) = 18, \quad f(-1) = -4$$

$$f(2) = 80, \quad f(-2) = 0$$

$\therefore (x+2)$  is a factor

$$\begin{array}{r} 3x^2 + 5x - 2 \\ x+2 \overline{) 3x^3 + 11x^2 + 8x - 4} \\ \underline{3x^3 + 6x^2} \phantom{- 4} \\ 5x^2 + 8x \phantom{- 4} \\ \underline{5x^2 + 10x} \phantom{- 4} \\ -2x - 4 \\ \underline{-2x - 4} \\ 0 \end{array}$$

$$\therefore f(x) = (x+2)(3x^2 + 5x - 2) \\ = (3x-1)(x+2)^2$$

$$\mathbf{b} \quad \frac{x+16}{f(x)} \equiv \frac{A}{3x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$x+16 \equiv A(x+2)^2 + B(3x-1)(x+2) + C(3x-1)$$

$$x = \frac{1}{3} \quad \Rightarrow \quad \frac{49}{3} = \frac{49}{9}A \quad \Rightarrow \quad A = 3$$

$$x = -2 \quad \Rightarrow \quad 14 = -7C \quad \Rightarrow \quad C = -2$$

$$\text{coeffs of } x^2 \Rightarrow 0 = A + 3B \Rightarrow B = -1$$

$$\therefore \frac{x+16}{f(x)} = \frac{3}{3x-1} - \frac{1}{x+2} - \frac{2}{(x+2)^2}$$

$$9 \quad \mathbf{a} \quad \frac{x+5}{(x-1)(2x+1)} \equiv \frac{A}{x-1} + \frac{B}{2x+1}$$

$$x+5 \equiv A(2x+1) + B(x-1)$$

$$x=1 \quad \Rightarrow \quad 6 = 3A \quad \Rightarrow \quad A = 2$$

$$x = -\frac{1}{2} \quad \Rightarrow \quad \frac{9}{2} = -\frac{3}{2}B \quad \Rightarrow \quad B = -3$$

$$\therefore f(x) = \frac{2}{x-1} - \frac{3}{2x+1}$$

$$\mathbf{b} \quad f'(x) = -2(x-1)^{-2} + 3(2x+1)^{-2} \times 2$$

$$= \frac{6}{(2x+1)^2} - \frac{2}{(x-1)^2}$$

$$\text{SP: } \frac{6}{(2x+1)^2} - \frac{2}{(x-1)^2} = 0$$

$$6(x-1)^2 - 2(2x+1)^2 = 0$$

$$x^2 + 10x - 2 = 0$$

$$x = \frac{-10 \pm \sqrt{100+8}}{2}$$

$$x = -5 \pm 3\sqrt{3}$$

$$8 \quad \mathbf{a} \quad \frac{4}{(x+1)(x-1)} \equiv \frac{A}{x+1} + \frac{B}{x-1}$$

$$4 \equiv A(x-1) + B(x+1)$$

$$x=-1 \quad \Rightarrow \quad 4 = -2A \quad \Rightarrow \quad A = -2$$

$$x=1 \quad \Rightarrow \quad 4 = 2B \quad \Rightarrow \quad B = 2$$

$$\therefore f(x) = \frac{2}{x-1} - \frac{2}{x+1}$$

$$\mathbf{b} \quad \frac{2+5x-x^2}{(x-4)(x-2)(x-1)} \equiv \frac{A}{x-4} + \frac{B}{x-2} + \frac{C}{x-1}$$

$$2+5x-x^2 \equiv A(x-2)(x-1) + B(x-4)(x-1) + C(x-4)(x-2)$$

$$x=4 \quad \Rightarrow \quad 6 = 6A \quad \Rightarrow \quad A = 1$$

$$x=2 \quad \Rightarrow \quad 8 = -2B \quad \Rightarrow \quad B = -4$$

$$x=1 \quad \Rightarrow \quad 6 = 3C \quad \Rightarrow \quad C = 2$$

$$\therefore g(x) = \frac{1}{x-4} - \frac{4}{x-2} + \frac{2}{x-1}$$

$$\mathbf{c} \quad \frac{2}{x-1} - \frac{2}{x+1} = \frac{1}{x-4} - \frac{4}{x-2} + \frac{2}{x-1}$$

$$\frac{4}{x-2} - \frac{1}{x-4} - \frac{2}{x+1} = 0$$

$$4(x-4)(x+1) - (x-2)(x+1) - 2(x-2)(x-4) = 0$$

$$x^2 + x - 30 = 0$$

$$(x+6)(x-5) = 0, \quad x = -6, 5$$

$$10 \quad \mathbf{a} \quad x(4x+5) \equiv A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

$$x=1 \quad \Rightarrow \quad 9 = 9A \quad \Rightarrow \quad A = 1$$

$$x=-2 \quad \Rightarrow \quad 6 = -3C \quad \Rightarrow \quad C = -2$$

$$\text{coeffs } x^2 \Rightarrow 4 = A + B \Rightarrow B = 3$$

$$\mathbf{b} \quad x = -1 \quad \therefore y = \frac{1}{2}$$

$$f(x) = (x-1)^{-1} + 3(x+2)^{-1} - 2(x+2)^{-2}$$

$$f'(x) = -(x-1)^{-2} - 3(x+2)^{-2} + 4(x+2)^{-3}$$

$$\text{grad} = -\frac{1}{4} - 3 + 4 = \frac{3}{4}$$

$$\therefore y - \frac{1}{2} = \frac{3}{4}(x+1)$$

$$4y - 2 = 3x + 3$$

$$3x - 4y + 5 = 0$$