

- 1 Prove, by counter-example, that each of the following statements is false.
- a For all positive real values of x , $\sqrt[3]{x} \leq x$. (2)
- b For all positive integer values of n , $(n^3 - n + 7)$ is prime. (2)
- 2 Use proof by contradiction to prove that $\sqrt{\pi}$ is irrational.
(You may assume that π is irrational). (4)
- 3 Find a counter-example to prove that the statement
“ $15x^2 - 11x + 2 \geq 0$ for all real values of x ”
is false. (4)
- 4 a Given that $n = 2m + 1$, find and simplify an expression in terms of m for $n^2 + 2n$. (1)
- b Hence, use proof by contradiction to prove that if $(n^2 + 2n)$ is even, where n is an integer, then n is even. (5)
- 5 a Prove that if the equation
 $k \cos x - \operatorname{cosec} x = 0$,
where k is a constant, has real solutions, then $|k| \geq 2$. (5)
- b Find the values of x in the interval $0 \leq x \leq 360$ for which
 $3 \cos x^\circ - \operatorname{cosec} x^\circ = 0$. (3)
- 6 Use proof by contradiction to prove that there are no positive integers, x and y , such that
 $x^2 - y^2 = 1$. (6)
- 7 For each statement, either prove that it is true or find a counter-example to prove that it is false.
- a If a and b are irrational and $a \neq b$, then $(a + b)$ is irrational. (2)
- b If m and n are consecutive odd integers, then $(m + n)$ is divisible by 4. (3)
- c For all real values of x , $\cos x \leq 1 + \sin x$. (2)
- 8 a Show that if $\log_2 3 = \frac{p}{q}$, then
 $2^p = 3^q$. (2)
- b Use proof by contradiction to prove that $\log_2 3$ is irrational. (4)
- c Prove, by counter-example, that the statement
“if a is rational and b is irrational then $\log_a b$ is irrational”
is false. (2)
- 9 The function f is defined by
 $f: x \rightarrow \frac{x-2}{4x}, x \in \mathbb{R}, x \neq 0$.
- a Find an expression for the inverse function, $f^{-1}(x)$, and state its domain. (4)
- b Prove that there are no real values of x for which
 $f(x) = f^{-1}(x)$. (4)