C3 NUMERICAL METHODS Worksheet C

1 **a** Show that the equation $x^3 - 7x - 11 = 0$ has a real root in the interval (3, 4).

b Using the iterative formula $x_{n+1} = \sqrt{7 + \frac{11}{x_n}}$, with $x_0 = 3.2$, find x_1, x_2 and x_3, giving the value of x_3 correct to 2 decimal places.

2 $$f(x) \equiv 4 \operatorname{cosec} x - 5 + 2x.$$

a Find the values of f(4) and f(5).

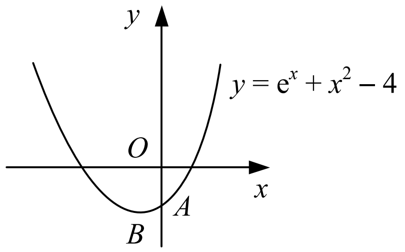
b Hence show that the equation $f(x) = 0$ has a root in the interval (4, 5).

The iterative formula $x_{n+1} = a + \dfrac{b}{\sin x_n}$, where a and b are constants, is used to find this root.

c Find the values of a and b.

d Starting with $x_0 = 4.5$, use the iterative formula with your values of a and b to find 3 further approximations of the root, giving your final answer correct to 3 decimal places.

3



The diagram shows the curve $y = e^x + x^2 - 4$. The curve intersects the y-axis at the point A and has a stationary point at B.

a Find $\dfrac{dy}{dx}$.

b Find an equation for the tangent to the curve at A.

c Show that the x-coordinate of B lies in the interval [–0.4, –0.3].

d Using an iteration process based on the rearrangement $x = \frac{1}{3}(x - e^x)$, with $x_0 = -0.3$, find the x-coordinate of B correct to 3 decimal places.

4 **a** On the same set of axes, sketch the curves $y = \cos x$ and $y = x^2$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

b Show that the equation $\cos x - x^2 = 0$ has exactly one positive and one negative real root.

c Show that the positive real root lies in the interval [0.8, 0.9].

d Use the iteration formula $x_{n+1} = \sqrt{\cos x_n}$ and the starting value $x_0 = 0.8$ to find the positive root correct to 2 decimal places.

5 $$f(x) \equiv e^{5-2x} - x^5.$$

Show that the equation $f(x) = 0$

a has a root in the interval (1.4, 1.5),

b can be written as $x = e^{1-kx}$, stating the value of k.

c Using the iteration formula $x_{n+1} = e^{1-kx_n}$, with $x_0 = 1.5$ and the value of k found in part **b**, find x_1, x_2 and x_3. Give the value of x_3 correct to 3 decimal places.

6 $\mathrm{f} : x \rightarrow 2^x + x^3 - 5, \; x \in \mathbb{R}.$

a Show that there is a solution of the equation $\mathrm{f}(x) = 0$ in the interval $1.3 < x < 1.4$

b Using the iterative formula $x_{n+1} = \sqrt[3]{5 - 2^{x_n}}$, with $x_0 = 1.4$, find x_1, x_2, x_3 and x_4.

c Hence write down an approximation for this solution of the equation $\mathrm{f}(x) = 0$ to an appropriate degree of accuracy.

Another attempt is made to find the solution using the iterative formula $x_{n+1} = \dfrac{\ln(5 - x_n^{\,3})}{\ln 2}$.

d Describe the outcome of this attempt.

7 $\mathrm{f}(x) = 2x^3 + 4x - 9.$

a Find $\mathrm{f}'(x)$.

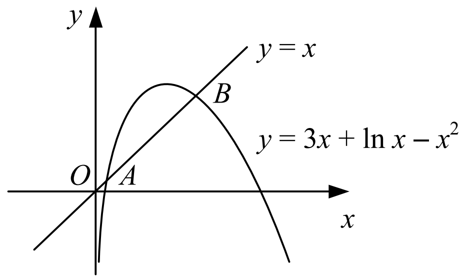
b Hence show that the equation $\mathrm{f}(x) = 0$ has exactly one real root.

c Show that this root lies in the interval $(1.2, 1.3)$.

d Use the iterative formula $x_{n+1} = \sqrt[3]{4.5 - 2x_n}$, with $x_0 = 1.2$, to find the root of $\mathrm{f}(x) = 0$ correct to 2 decimal places.

e Justify the accuracy of your answer.

8



The diagram shows part of the curve with equation $y = 3x + \ln x - x^2$ and the line $y = x$.

Given that the curve and line intersect at the points A and B, show that

a the x-coordinates of A and B are the solutions of the equation $x = \mathrm{e}^{x^2 - 2x}$,

b the x-coordinate of A lies in the interval $(0.4, 0.5)$,

c the x-coordinate of B lies in the interval $(2.3, 2.4)$.

d Use the iteration formula $x_{n+1} = \mathrm{e}^{x_n^{\,2} - 2x_n}$, with $x_0 = 0.5$, to find the x-coordinate of A correct to 2 decimal places.

e Justify the accuracy of your answer to part **d**.

9 **a** On the same set of axes, sketch the graphs of $y = x^4$ and $y = 5x + 2$.

b Show that the equation $x^4 - 5x - 2 = 0$ has exactly one positive and one negative real root.

c Use the iteration formula $x_{n+1} = \sqrt[4]{5x_n + 2}$, with $x_0 = 1.8$, to find x_1, x_2, x_3 and x_4, giving the value of x_4 correct to 3 decimal places.

d Show that the equation $x^4 - 5x - 2 = 0$ can be written in the form $x = \dfrac{a}{x^3 + b}$, stating the values of a and b.

e Use the iteration formula $x_{n+1} = \dfrac{a}{x_n^{\,3} + b}$, with $x_0 = -0.4$ and your values of a and b, to find the negative real root of the equation correct to 4 decimal places.