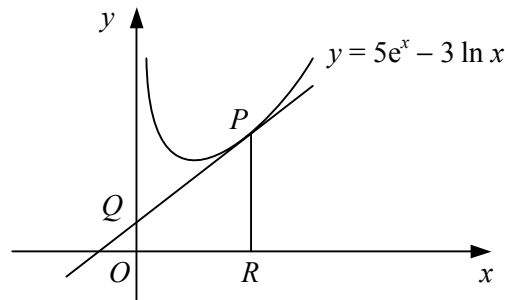


- 1 a Find an equation for the normal to the curve $y = \frac{2}{5}x + \frac{1}{10}e^x$ at the point on the curve where $x = 0$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.
b Find the coordinates of the point where this normal crosses the x -axis.

2



The diagram shows the curve with equation $y = 5e^x - 3 \ln x$ and the tangent to the curve at the point P with x -coordinate 1.

- a Show that the tangent at P has equation $y = (5e - 3)x + 3$.

The tangent at P meets the y -axis at Q .

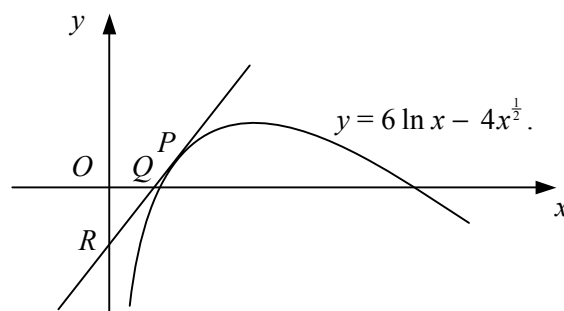
The line through P parallel to the y -axis meets the x -axis at R .

- b Find the area of trapezium $ORPQ$, giving your answer in terms of e .

3 A curve has equation $y = 3x - \frac{1}{2}e^x$.

- a Find the coordinates of the stationary point on the curve, giving your answers in terms of natural logarithms.
b Determine the nature of the stationary point.

4



The diagram shows the curve $y = 6 \ln x - 4x^{\frac{1}{2}}$. The x -coordinate of the point P on the curve is 4. The tangent to the curve at P meets the x -axis at Q and the y -axis at R .

- a Find an equation for the tangent to the curve at P .
b Hence, show that the area of triangle OQR is $(10 - 12 \ln 2)^2$.

5 The curve with equation $y = 2x - 2 - \ln x$ passes through the point $A(1, 0)$. The tangent to the curve at A crosses the y -axis at B and the normal to the curve at A crosses the y -axis at C .

- a Find an equation for the tangent to the curve at A .
b Show that the mid-point of BC is the origin.

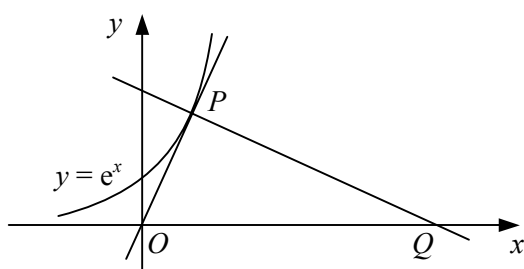
The curve has a minimum point at D .

- c Show that the y -coordinate of D is $\ln 2 - 1$.

- 6 a Sketch the curve with equation $y = e^x + k$, where k is a positive constant.
Show on your sketch the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes.
- b Find an equation for the tangent to the curve at the point on the curve where $x = 2$.
Given that the tangent passes through the x -axis at the point $(-1, 0)$,
- c show that $k = 2e^2$.

- 7 A curve has equation $y = 3x^2 - 2 \ln x$, $x > 0$.
The gradient of the curve at the point P on the curve is -1 .
- a Find the x -coordinate of P .
- b Find an equation for the tangent to the curve at the point on the curve where $x = 1$.

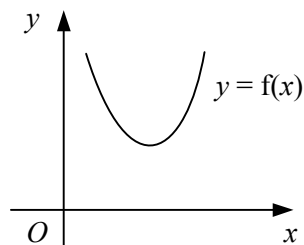
8



The diagram shows the curve with equation $y = e^x$ which passes through the point $P(p, e^p)$.
Given that the tangent to the curve at P passes through the origin and that the normal to the curve at P meets the x -axis at Q ,

- a show that $p = 1$,
- b show that the area of triangle OPQ , where O is the origin, is $\frac{1}{2}e(1 + e^2)$.
- 9 The curve with equation $y = 4 - e^x$ meets the y -axis at the point P and the x -axis at the point Q .
- a Find an equation for the normal to the curve at P .
- b Find an equation for the tangent to the curve at Q .
The normal to the curve at P meets the tangent to the curve at Q at the point R .
The x -coordinate of R is $a \ln 2 + b$, where a and b are rational constants.
- c Show that $a = \frac{8}{5}$.
- d Find the value of b .

10



The diagram shows a sketch of the curve $y = f(x)$ where

$$f: x \rightarrow 9x^4 - 16 \ln x, \quad x > 0.$$

Given that the set of values of x for which $f(x)$ is a decreasing function of x is $0 < x \leq k$, find the exact value of k .