1 **a** $\frac{2}{\cos x} = \frac{3}{\sin x}$

$\frac{\sin x}{\cos x} = \frac{3}{2}$

$\tan x = \frac{3}{2}$

$x = 56.3,\ 56.3 - 180$

$x = -123.7°,\ 56.3°$

b $\cot^2\theta - \cot\theta + 1 + \cot^2\theta = 4$

$2\cot^2\theta - \cot\theta - 3 = 0$

$(2\cot\theta - 3)(\cot\theta + 1) = 0$

$\cot\theta = -1$ or $\frac{3}{2}$

$\tan\theta = -1$ or $\frac{2}{3}$

$\theta = \pi - \frac{\pi}{4},\ 2\pi - \frac{\pi}{4}$ or $0.5880,\ \pi + 0.5880$

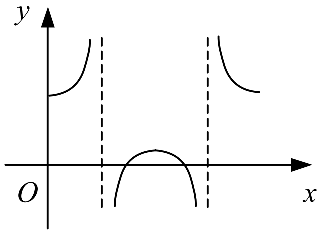
$\theta = 0.588$ (3sf), $\frac{3\pi}{4}$, 3.73 (3sf), $\frac{7\pi}{4}$

2 **a** LHS $= 4\sin^2\theta - 4 + \text{cosec}^2\theta$

$= 4(1 - \cos^2\theta) - 4 + \text{cosec}^2\theta$

$= \text{cosec}^2\theta - 4\cos^2\theta =$ RHS

b **i**



ii (0, 5)

iii $3 + 2\sec x = 0$

$\sec x = -\frac{3}{2}$, $\cos x = -\frac{2}{3}$

$x = \pi - 0.841,\ \pi + 0.841 = 2.30,\ 3.98$

$\therefore$ (2.30, 0) and (3.98, 0) [x to 3sf]

3 **a** **i** $\text{cosec}\,A = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{2+\sqrt{3}}{4-3} = 2 + \sqrt{3}$

ii $\text{cosec}^2 A = (2 + \sqrt{3})^2$

$= 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}$

$\cot^2 A = \text{cosec}^2 A - 1 = 6 + 4\sqrt{3}$

b $3(1 - 2\sin^2 x) - 8\sin x + 5 = 0$

$3\sin^2 x + 4\sin x - 4 = 0$

$(3\sin x - 2)(\sin x + 2) = 0$

$\sin x = \frac{2}{3}$ or -2 [no solutions]

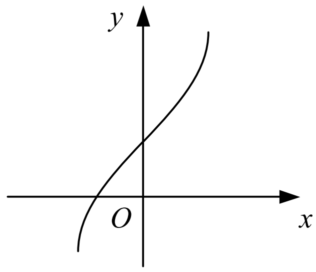
$x = 41.8,\ 180 - 41.8$

$x = 41.8°,\ 138.2°$

4 **a** $= \frac{\pi}{2} + 2 \times \frac{\pi}{6} = \frac{5\pi}{6}$

b $-\frac{\pi}{2} \le f(x) \le \frac{3\pi}{2}$

c



d $\frac{\pi}{2} + 2\sin^{-1} x = 0$

$\sin^{-1} x = -\frac{\pi}{4}$

$x = \sin(-\frac{\pi}{4}) = -\frac{1}{\sqrt{2}}$

5 **a** $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$

let $A = B = \frac{x}{2}$

$\cos x \equiv \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$

$\cos x \equiv \cos^2 \frac{x}{2} - (1 - \cos^2 \frac{x}{2})$

$\cos x \equiv 2\cos^2 \frac{x}{2} - 1$

b $\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{1+(2\cos^2\frac{x}{2}-1)} = 3\cot\frac{x}{2}$

$\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} = 3\cot\frac{x}{2}$

$\tan\frac{x}{2} = \frac{3}{\tan\frac{x}{2}}$

$\tan^2\frac{x}{2} = 3$

$\tan\frac{x}{2} = \pm\sqrt{3}$

$\frac{x}{2} = 60$ or $180 - 60$

$\frac{x}{2} = 60, 120$

$x = 120°, 240°$

6 **a** $3\cos\theta + 4\sin\theta$

$= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$\Rightarrow R\cos\alpha = 3,\ R\sin\alpha = 4$

$\therefore R = \sqrt{9+16} = 5$

$\tan\alpha = \frac{4}{3},\ \alpha = 0.927$ (3sf)

$\therefore 3\cos\theta + 4\sin\theta = 5\cos(\theta - 0.927)$

b **i** $-4 \le f(\theta) \le 6$

ii $1 - 5\cos(2\theta - 0.9273) = 0$

$\cos(2\theta - 0.9273) = \frac{1}{5}$

$2\theta - 0.9273 = 1.3694,\ 2\pi - 1.3694$

$= 1.3694,\ 4.9137$

$2\theta = 2.2967,\ 5.8410$

$\theta = 1.15,\ 2.92$ (3sf)

c $y = \frac{2}{5\cos(x - 0.9273)}$

TP: $y = \frac{2}{5}$ when $x - 0.9273 = 0$

$y = -\frac{2}{5}$ when $x - 0.9273 = \pi$

$\therefore (0.927, \frac{2}{5})$ and $(4.07, -\frac{2}{5})$ [x to 3sf]