1

$\sec^2 x - 1 - \sec x = 1$

$\sec^2 x - \sec x - 2 = 0$

$(\sec x + 1)(\sec x - 2) = 0$

$\sec x = -1$ or 2

$\cos x = -1$ or $\frac{1}{2}$

$x = 180$ or $60, 360 - 60$

$x = 60°, 180°, 300°$

2 **a** $2 \cos x + 5 \sin x$

$= R \cos x \cos \alpha + R \sin x \sin \alpha$

$\Rightarrow R \cos \alpha = 2, \ R \sin \alpha = 5$

$\therefore R = \sqrt{4+25} = \sqrt{29} = 5.39$ (3sf)

$\tan \alpha = \frac{5}{2}, \ \alpha = 68.2$ (3sf)

$\therefore 2 \cos x + 5 \sin x = 5.39 \cos (x - 68.2)°$

b $\sqrt{29} \cos (x - 68.199) = 3$

$\cos (x - 68.199) = \frac{3}{\sqrt{29}} = 0.5571$

$x - 68.199 = 56.145, -56.145$

$x = 12.1, 124.3$

3

$2 \sin \theta \cos 30 + 2 \cos \theta \sin 30$

$= \sin \theta \cos 30 - \cos \theta \sin 30$

$\sqrt{3} \sin \theta + \cos \theta = \frac{\sqrt{3}}{2} \sin \theta - \frac{1}{2} \cos \theta$

$\frac{\sqrt{3}}{2} \sin \theta = -\frac{3}{2} \cos \theta$

$\frac{\sin\theta}{\cos\theta} = -\sqrt{3}$

$\tan \theta = -\sqrt{3}$

$\theta = 180 - 60, 360 - 60$

$\theta = 120°, 300°$

4 **a** $\tan^{-1} 2x = \frac{\pi}{6}$

$2x = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$x = \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{6}\sqrt{3}$

b $4 \sin x \cos x = 3 \cos x$

$\cos x(4 \sin x - 3) = 0$

$\cos x = 0$ or $\sin x = \frac{3}{4}$

$x = 90, 360 - 90$ or $48.6, 180 - 48.6$

$x = 48.6°$ (3sf), $90°$, $131°$ (3sf), $270°$

5 **a** LHS $= \sec x + \tan x - \tan x - \sin x \tan x$

$= \frac{1}{\cos x} - \sin x \times \frac{\sin x}{\cos x}$

$= \frac{1 - \sin^2 x}{\cos x}$

$= \frac{\cos^2 x}{\cos x}$

$= \cos x$

$=$ RHS

b $2(1 + \tan^2 2y) + \tan^2 2y = 3$

$\tan^2 2y = \frac{1}{3}$

$\tan 2y = \pm\frac{1}{\sqrt{3}}$

$2y = \frac{\pi}{6}, \pi + \frac{\pi}{6}$ or $\pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$y = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$

6 **a** $4 \sin x - \cos x$

$= R \sin x \cos \alpha - R \cos x \sin \alpha$

$\Rightarrow R \cos \alpha = 4, \ R \sin \alpha = 1$

$\therefore R = \sqrt{16+1} = \sqrt{17} = 4.12$ (3sf)

$\tan \alpha = \frac{1}{4}, \alpha = 14.0$ (3sf)

$\therefore 4 \sin x° - \cos x° = 4.12 \sin (x - 14.0)°$

b $\frac{2}{\sin x} - \frac{\cos x}{\sin x} + 4 = 0$

$2 - \cos x + 4 \sin x = 0$

$\therefore 4 \sin x° - \cos x° + 2 = 0$

c $\sqrt{17} \sin (x - 14.04) + 2 = 0$

$\sin (x - 14.04) = -\frac{2}{\sqrt{17}}$

$x - 14.04 = 180 + 29.02, 360 - 29.02$

$= 209.02, 330.98$

$x = 223, 345$ (3sf)

7 **a** LHS = $\frac{1}{\sin\theta} - \sin\theta$

$= \frac{1-\sin^2\theta}{\sin\theta}$

$= \frac{\cos^2\theta}{\sin\theta}$

$= \cos\theta \times \frac{\cos\theta}{\sin\theta}$

$= \cos\theta\cot\theta$

= RHS

b $\frac{2}{\cos x} + \frac{\sin x}{\cos x} = 2\cos x$

$2 + \sin x = 2\cos^2 x$

$2 + \sin x = 2(1 - \sin^2 x)$

$2\sin^2 x + \sin x = 0$

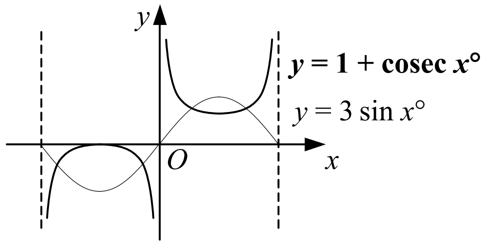
$\sin x(2\sin x + 1) = 0$

$\sin x = -\frac{1}{2}$ or 0

$x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$ or $0, \pi, 2\pi$

$x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$

8 **a**



b $3\sin x = 1 + \frac{1}{\sin x}$

$3\sin^2 x - \sin x - 1 = 0$

$\sin x = \frac{1 \pm \sqrt{1+12}}{6} = \frac{1 \pm \sqrt{13}}{6}$

$\sin x = -0.4343$ or 0.7676

$x = -25.7, 25.7 - 180$ or $50.1, 180 - 50.1$

$x = -154.3, -25.7, 50.1, 129.9$

9 **a** LHS = $\frac{\cos 2x}{\sin 2x} + \frac{1}{\sin 2x}$

$= \frac{\cos 2x + 1}{\sin 2x}$

$= \frac{2\cos^2 x - 1 + 1}{2\sin x\cos x}$

$= \frac{2\cos^2 x}{2\sin x\cos x}$

$= \frac{\cos x}{\sin x}$

$= \cot x$ = RHS

b $\cot x = 6 - \cot^2 x$

$\cot^2 x + \cot x - 6 = 0$

$(\cot x + 3)(\cot x - 2) = 0$

$\cot x = -3$ or 2

$\tan x = -\frac{1}{3}$ or $\frac{1}{2}$

$x = \pi - 0.3218, 2\pi - 0.3218$

or $0.4636, \pi + 0.4636$

$x = 0.46, 2.82, 3.61, 5.96$

10 **a** LHS = $\cos x\cos 30 - \sin x\sin 30 + \sin x$

$= \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x + \sin x$

$= \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$

$= \cos x\cos 30 + \sin x\sin 30$

$= \cos(x - 30)°$ = RHS

b let $x = 45$

$\cos 75° + \sin 45° = \cos 15°$

$\therefore \cos 75° - \cos 15° = -\sin 45°$

$= -\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = -\frac{1}{2}\sqrt{2}$

c $3\cos(x + 30) + 3\sin x - 2\sin x$

$= 3\cos(x - 30) + 1$

$-2\sin x = 1$

$\sin x = -\frac{1}{2}$

$x = -30, 30 - 180$

$x = -150, -30$

11 **a** $a = 3$

$b \sin x° + c \cos x°$ can be expressed in the form $k \sin (x + \alpha)°$ which will vary between $-k$ and $+k$

$\therefore\ a + k = 5$ and $a - k = 1$, hence $a = 3$

b $3 + k = 5\ \therefore\ k = 2$

$60 + \alpha = 90\ \therefore\ \alpha = 30$

c $f(x) = 3 + 2 \sin (x + 30)$

$= 3 + 2 \sin x \cos 30 + 2 \cos x \sin 30$

$= 3 + \sqrt{3} \sin x + \cos x$

$\therefore\ b = \sqrt{3},\ c = 1$

12 **a** LHS $= \dfrac{1-(1-2\sin^2 \frac{x}{2})}{1+(2\cos^2 \frac{x}{2}-1)}$

$= \dfrac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}}$

$= \tan^2 \dfrac{x}{2}$ = RHS

b **i** let $x = \frac{\pi}{6}$, $\dfrac{1-\cos\frac{\pi}{6}}{1+\cos\frac{\pi}{6}} = \tan^2 \frac{\pi}{12}$

$\tan^2 \frac{\pi}{12} = \dfrac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}} = \dfrac{2-\sqrt{3}}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \dfrac{4-4\sqrt{3}+3}{4-3} = 7 - 4\sqrt{3}$

ii $\tan^2 \dfrac{x}{2} = 1 - \sec \dfrac{x}{2}$

$\sec^2 \dfrac{x}{2} - 1 = 1 - \sec \dfrac{x}{2}$

$\sec^2 \dfrac{x}{2} + \sec \dfrac{x}{2} - 2 = 0$

$(\sec \dfrac{x}{2} + 2)(\sec \dfrac{x}{2} - 1) = 0$

$\sec \dfrac{x}{2} = -2$ or 1

$\cos \dfrac{x}{2} = -\frac{1}{2}$ or 1

$\dfrac{x}{2} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ or 0

$x = 0,\ \frac{4\pi}{3}$

13 **a** $2 \sin x - 3 \cos x$

$= R \sin x \cos \alpha - R \cos x \sin \alpha$

$\Rightarrow\ R \cos \alpha = 2,\ R \sin \alpha = 3$

$\therefore\ R = \sqrt{4+9} = \sqrt{13} = 3.61$

$\tan \alpha = \frac{3}{2},\ \alpha = 0.983$

$\therefore\ 2 \sin x - 3 \cos x = 3.61 \sin (x - 0.983)$

b min. value $= -3.61$ (3sf)

when $x - 0.9828 = \frac{3\pi}{2},\ x = 5.70$ (3sf)

c $\sqrt{13} \sin (2x - 0.9828) + 1 = 0$

$\sin (2x - 0.9828) = -\dfrac{1}{\sqrt{13}}$

$2x - 0.983 = \pi + 0.2810, -0.2810$

$= -0.2810, 3.4226$

$2x = 0.7018, 4.4054$

$x = 0.35, 2.20$