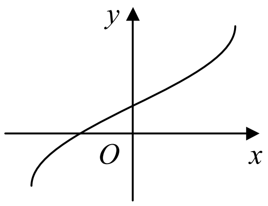
1 **a**



b $-\frac{\pi}{3} \leq \text{f}(x) \leq \frac{2\pi}{3}$

c $(-1, 0)$, $(0, \frac{\pi}{6})$

d

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
f(x)	0.5236	0.7763	1.0472	1.3717	2.0944

$\therefore$ area $\approx \frac{1}{3} \times \frac{1}{2} \times [0.5236 + 2.0944 + 4(0.7763 + 1.3717) + 2(1.0472)] = 2.22$ (3sf)

2 **a** at A, $x^{\frac{5}{2}} - 3x^{\frac{1}{2}} - 7x = 0$

let $\text{f}(x) = x^{\frac{5}{2}} - 3x^{\frac{1}{2}} - 7x$

$\text{f}(4) = -2$

$\text{f}(5) = 14.2$

sign change, f(x) continuous $\therefore$ root

$\therefore\ 4 < \alpha < 5$

b $\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7$

at B, $\frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7 = 0$

let $\text{g}(x) = \frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7$

$\text{g}(2) = -0.990$

$\text{g}(3) = 5.12$

sign change, g(x) continuous $\therefore$ root

$\therefore\ 2 < \beta < 3$

c $\frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{-\frac{1}{2}} - 7 = 0$

$5x^2 - 3 - 14x^{\frac{1}{2}} = 0$

$x^2 = 0.6 + 2.8x^{\frac{1}{2}}$

$x > 0 \therefore x = \beta$ is a soln to $x = \sqrt{0.6 + 2.8x^{\frac{1}{2}}}$

d $x_1 = 2.158144$

$x_2 = 2.171031$

$x_3 = 2.173853$

$x_4 = 2.174470$

$x_5 = 2.174604$

$\therefore\ \beta = 2.175$ (4sf)

3 **a** f(4) = 1.28
f(5) = –2.92
sign change, f(x) continuous ∴ root

b $x_{n+1} = 3 + \ln(2x_n^{\frac{1}{2}})$
$x_1 = 4.445186$
$x_2 = 4.439058$
$x_3 = 4.438368$
$x_4 = 4.438291$
∴ root = 4.438 (3dp)

c f(4.4375) = 0.00292
f(4.4385) = –0.000820
sign change, f(x) continuous ∴ root

d

x	1	2.5	4
f(x)	1.8647	2.5557	1.2817

∴ integral ≈ $\frac{1}{3} \times 1.5 \times [1.8647 + 1.2817 + 4(2.5557)] = 6.68$ (3sf)

4 **a** $\frac{dy}{dx} = 3 - \frac{1}{x}$
grad = 2
∴ grad of normal = $-\frac{1}{2}$
∴ $y - 3 = -\frac{1}{2}(x - 1)$
$[\, y = \frac{7}{2} - \frac{1}{2}x \,]$

b $3x - \ln x = \frac{7}{2} - \frac{1}{2}x$
$6x - 2\ln x = 7 - x$
$2\ln x - 7x + 7 = 0$

c $2\ln x = 7x - 7$
$\ln x = \frac{7}{2}(x - 1)$
$x = e^{\frac{7}{2}(x-1)}$ ∴ $k = \frac{7}{2}$

d $x_1 = 0.173774$
$x_2 = 0.055477$
$x_3 = 0.036669$
$x_4 = 0.034333$
$x_5 = 0.034053$
∴ x-coord of Q = 0.034 (3dp)

e let $f(x) = 2\ln x - 7x + 7$
f(0.0335) = –0.027
f(0.0345) = 0.025
sign change, f(x) continuous ∴ root