

1 a  $9 + 4x - 2x^3 = 0$

$$x^3 = 2x + 4.5$$

$$x = \sqrt[3]{2x + 4.5}$$

$$\therefore x_{n+1} = \sqrt[3]{2x_n + 4.5}$$

$$x_1 = 2.040828$$

$$x_2 = 2.047342$$

$$x_3 = 2.048377 = 2.0484 \text{ (4dp)}$$

c  $\tan x - 5x + 13 = 0$

$$\tan x = 5x - 13$$

$$x = \tan^{-1}(5x - 13)$$

$$\therefore x_{n+1} = \tan^{-1}(5x_n - 13)$$

$$x_1 = -1.518213$$

$$x_2 = -1.522270$$

$$x_3 = -1.522317 = -1.5223 \text{ (4dp)}$$

2 a  $e^{2x-1} - 6x = 0$

$$e^{2x-1} = 6x$$

$$2x - 1 = \ln 6x$$

$$x = \frac{1}{2}(\ln 6x + 1)$$

$$\therefore x_{n+1} = \frac{1}{2}(\ln 6x_n + 1), a = \frac{1}{2}, b = 6$$

$$x_1 = 1.661194$$

$$x_2 = 1.649648$$

$$x_3 = 1.646161 = 1.646 \text{ (3dp)}$$

c  $2x^3 - 6x - 11 = 0$

$$2x^3 = 6x + 11$$

$$x^2 = 3 + \frac{11}{2x}$$

$$x = \pm \sqrt{3 + \frac{11}{2x}}$$

$$\therefore x_{n+1} = \sqrt{3 + \frac{5.5}{x_n}}, a = 3, b = 5.5$$

$$x_1 = 2.397916$$

$$x_2 = 2.300795$$

$$x_3 = 2.321740 = 2.322 \text{ (3dp)}$$

b  $e^x - 8x + 5 = 0$

$$e^x = 8x - 5$$

$$x = \ln(8x - 5)$$

$$\therefore x_{n+1} = \ln(8x_n - 5)$$

$$x_1 = 2.944439$$

$$x_2 = 2.920767$$

$$x_3 = 2.910508 = 2.9105 \text{ (4dp)}$$

d  $\ln x + \sqrt{x} + 1.4 = 0$

$$\ln x = -(\sqrt{x} + 1.4)$$

$$x = e^{-(\sqrt{x} + 1.4)}$$

$$\therefore x_{n+1} = e^{-(\sqrt{x_n} + 1.4)}$$

$$x_1 = 0.165299$$

$$x_2 = 0.164216$$

$$x_3 = 0.164436 = 0.1644 \text{ (4dp)}$$

b  $\frac{2}{x} + \cos x - 3 = 0$

$$\frac{2}{x} = 3 - \cos x$$

$$2 = x(3 - \cos x)$$

$$x = \frac{2}{3 - \cos x}$$

$$\therefore x_{n+1} = \frac{2}{3 - \cos x_n}, a = 2, b = 3$$

$$x_1 = 0.868322$$

$$x_2 = 0.849657$$

$$x_3 = 0.854789 = 0.855 \text{ (3dp)}$$

d  $15 \ln(x + 3) - 4x = 0$

$$\ln(x + 3) = \frac{4}{15}x$$

$$x + 3 = e^{\frac{4}{15}x}$$

$$x = e^{\frac{4}{15}x} - 3$$

$$\therefore x_{n+1} = e^{\frac{4}{15}x_n} - 3, a = \frac{4}{15}, b = -3$$

$$x_1 = -2.486583$$

$$x_2 = -2.484743$$

$$x_3 = -2.484490 = -2.484 \text{ (3dp)}$$

- 3 a**  $x_1 = 0.428135$   
 $x_2 = 0.433865$   
 $x_3 = 0.431107$   
 $x_4 = 0.432437$   
 $x_5 = 0.431796$   
 $\therefore$  root = 0.432 (3dp)  
 $f(0.4315) = -0.00465$   
 $f(0.4325) = 0.00457$   
 sign change,  $f(x)$  continuous  $\therefore$  root
- b**  $x_1 = 0.474342$   
 $x_2 = 0.470474$   
 $x_3 = 0.469923$   
 $\therefore$  root = 0.47 (2sf)  
 $f(0.465) = -0.00428$   
 $f(0.475) = 0.00463$   
 sign change,  $f(x)$  continuous  $\therefore$  root
- c**  $x_1 = 5.892685$   
 $x_2 = 5.859202$   
 $x_3 = 5.850013$   
 $x_4 = 5.847607$   
 $x_5 = 5.846985$   
 $x_6 = 5.846825$   
 $\therefore$  root = 5.85 (3sf)  
 $f(5.845) = 0.00658$   
 $f(5.855) = -0.0305$   
 sign change,  $f(x)$  continuous  $\therefore$  root
- d**  $x_1 = 3.731246$   
 $x_2 = 3.724839$   
 $x_3 = 3.726145$   
 $x_4 = 3.725879$   
 $\therefore$  root = 3.726 (3dp)  
 $f(3.7255) = 0.000672$   
 $f(3.7265) = -0.000912$   
 sign change,  $f(x)$  continuous  $\therefore$  root
- 4 a**  $x_{n+1} = \sqrt[5]{10x_n^3 - 4}$   
 $x_1 = -3.192595$   
 $x_2 = -3.188214$   
 $x_3 = -3.185620$   
 $x_4 = -3.184084$   
 $x_5 = -3.183174$   
 $x_6 = -3.182635$   
 $\therefore$  root = -3.18 (2dp)
- b**  $x^5 - 10x^3 + 4 = 0$   
 $4 = 10x^3 - x^5 = x^3(10 - x^2)$   
 $x^3 = \frac{4}{10 - x^2}$   
 $x = \sqrt[3]{\frac{4}{10 - x^2}} \therefore a = 4, b = 10$
- c**  $x_1 = 0.763143$   
 $x_2 = 0.751692$   
 $x_3 = 0.751231$   
 $x_4 = 0.751212$   
 $\therefore$  root = 0.751 (3dp)
- 5 a**  $\sin^{-1} 2x - 0.5x - 0.7 = 0$   
 $\sin^{-1} 2x = 0.5x + 0.7$   
 $2x = \sin(0.5x + 0.7)$   
 $x = 0.5 \sin(0.5x + 0.7) \therefore a = 0.5, b = 0.5, c = 0.7$
- b**  $x_1 = 0.391663$   
 $x_2 = 0.390365$   
 $x_3 = 0.390162$   
 $x_4 = 0.390130$   
 $\therefore$  solution = 0.390 (3dp)