1 **a** $f(1) = 2 + \log_4 1 = 2$
$ff(1) = f(2) = 2 + \log_4 2$
$= 2 + \frac{1}{2} = \frac{5}{2}$

b $2 + \log_4 x = 0$
$\log_4 x = -2$
$x = 4^{-2} = \frac{1}{16}$

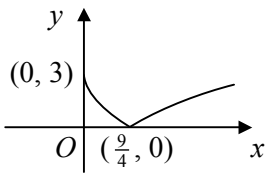
c $y = 2 + \log_4 x$
swap $x = 2 + \log_4 y$
$\log_4 y = x - 2$
$y = 4^{x-2}$
$f^{-1}(x) = 4^{x-2},\ x \in \mathbb{R}$

2 **a** $f(x) \in \mathbb{R}$
$g(x) \geq 0$

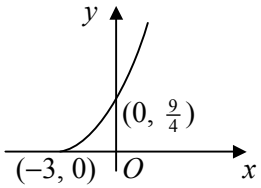
b $y = 2 + \ln 3x$
swap $x = 2 + \ln 3y$
$\ln 3y = x - 2$
$y = \frac{1}{3}e^{x-2}$
$f^{-1}(x) = \frac{1}{3}e^{x-2},\ x \in \mathbb{R}$

c $gf(x) = |2 + \ln 3x - 4| = |\ln 3x - 2|$
$\therefore\ |\ln 3x - 2| = 3$
$\ln 3x = 2 \pm 3 = -1, 5$
$x = \frac{1}{3}e^{-1}, \frac{1}{3}e^{5}$

3 **a** **i**



ii



b $f(0) = b = -3$
$f(\frac{9}{4}) = \frac{3}{2}a - 3 = 0$
$\therefore\ a = 2, b = -3$

c $y = 2x^{\frac{1}{2}} - 3$
swap $x = 2y^{\frac{1}{2}} - 3$
$y^{\frac{1}{2}} = \frac{1}{2}(x + 3)$
$f^{-1}(x) = \frac{1}{4}(x + 3)^2,\ x \in \mathbb{R},\ x \geq -3$

4 **a** $ff(x) = f\left(\frac{x+2}{x-1}\right) = \frac{\frac{x+2}{x-1} + 2}{\frac{x+2}{x-1} - 1}$
$= \frac{x + 2 + 2(x-1)}{x + 2 - (x-1)}$
$= \frac{3x}{3} = x$

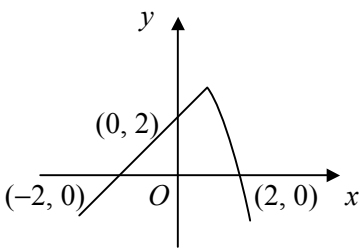
b $f^{-1}(x) = \frac{x+2}{x-1},\ x \in \mathbb{R},\ x \neq 1$

c $gf(x) = 2f(x) - 3 = 0$
$\therefore\ f(x) = \frac{x+2}{x-1} = \frac{3}{2}$
$2(x + 2) = 3(x - 1)$
$2x + 4 = 3x - 3$
$x = 7$

5 **a** $5 - 7 < 2x < 5 + 7$
$-2 < 2x < 12$
$-1 < x < 6$

b $(3x + 2)^2 = (3 - 2x)^2$
$9x^2 + 12x + 4 = 9 - 12x + 4x^2$
$5x^2 + 24x - 5 = 0$
$(5x - 1)(x + 5) = 0$
$x = -5, \frac{1}{5}$

6 **a**



b $f(3) = 4 - 9 = -5$
$ff(3) = f(-5) = -5 + 2 = -3$

c $x + 2 = 1 \Rightarrow x = -1$
$4 - x^2 = 1 \Rightarrow x^2 = 3,\quad x \geq 1\ \ \therefore x = \sqrt{3}$
$\therefore\ x = -1, \sqrt{3}$

7 **a** **i** $y = kx + 2$

swap $x = ky + 2$

$$y = \frac{x-2}{k}$$

$$f^{-1}(x) = \frac{x-2}{k},\ x \in \mathbb{R}$$

ii $fg(x) = f(x - 3k)$

$= k(x - 3k) + 2$

$fg(x) = kx - 3k^2 + 2,\ x \in \mathbb{R}$

b $4 = 7k - 3k^2 + 2$

$3k^2 - 7k + 2 = 0$

$(3k - 1)(k - 2) = 0$

$k = \frac{1}{3}, 2$

8 **a** $f(x) = (x - 2)^2 - 4 + 5 = (x - 2)^2 + 1$

b $f(x) \geq 1$

c $y = (x - 2)^2 + 1$

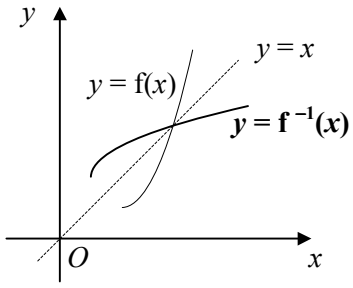
swap $x = (y - 2)^2 + 1$

$$y = 2 \pm \sqrt{x-1}$$

(domain of f $\Rightarrow$ +)

$f^{-1}(x) = 2 + \sqrt{x-1},\ x \in \mathbb{R},\ x \geq 1$

d



the graphs are reflections of each other in the line $y = x$

9 **a** $f(-2) = 8$

$gf(-2) = g(8) = 15\frac{7}{8}$

b $fg(x) = f(2x - \frac{1}{x}) = (2x - \frac{1}{x})^2 + 4$

$= 4x^2 - 4 + \frac{1}{x^2} + 4$

$fg(x) = 4x^2 + \frac{1}{x^2}$

c $4x^2 + \frac{1}{x^2} = 5$

$4x^4 - 5x^2 + 1 = 0$

$(4x^2 - 1)(x^2 - 1) = 0$

$x^2 = \frac{1}{4}, 1$

$x = \pm\frac{1}{2}, \pm 1$

10 **a** $y = e^{\frac{1}{2}x} - 3$

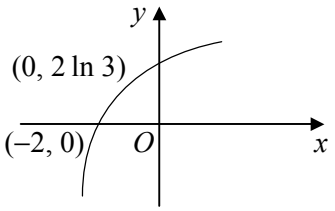
swap $x = e^{\frac{1}{2}y} - 3$

$$\tfrac{1}{2}y = \ln(x + 3)$$

$$y = 2\ln(x + 3)$$

$f^{-1}(x) = 2\ln(x + 3),\ x \in \mathbb{R},\ x > -3$

b



c $g(4) = \ln 9 = 2\ln 3$

$fg(4) = f(2\ln 3) = 0$

d $2\ln(x + 3) = \ln(x + 5)$

$(x + 3)^2 = x + 5$

$x^2 + 5x + 4 = 0$

$(x + 4)(x + 1) = 0$

$x > -3 \ \therefore x = -1$