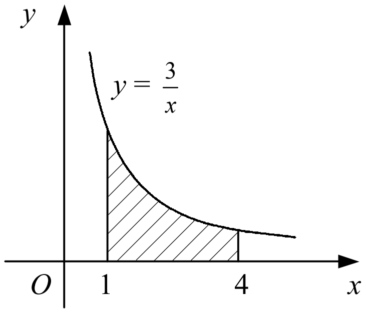
C2 INTEGRATION Worksheet G

1



The diagram shows the curve with equation $y = \frac{3}{x}$, $x > 0$.

a Copy and complete the table below, giving the exact y-coordinate corresponding to each x-coordinate for points on the curve.

x	1	2	3	4
y				

The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$.

b Use the trapezium rule with all the values in your table to show that the area of the shaded region is approximately $4\frac{3}{8}$.

c With the aid of a sketch diagram, explain whether the true area is more or less than $4\frac{3}{8}$.

2 **a** Sketch the curve $y = x(3x + 2)$ showing the coordinates of any points of intersection with the coordinate axes.

b Use the trapezium rule with four intervals of equal width to estimate the area bounded by the curve, the x-axis and the line $x = 2$.

c Find this area exactly using integration.

d Hence, find the percentage error in the estimate made in part **b**.

3 Use the trapezium rule with the stated number of intervals of equal width to estimate the area of the region enclosed by the given curve, the x-axis and the given ordinates.

a $y = \frac{3}{2x+1}$ $\quad x = 4 \quad x = 6 \quad$ 2 intervals

b $y = \lg(x^2 + 9)$ $\quad x = 0 \quad x = 3 \quad$ 3 intervals

c $y = x^2 \sin x$ $\quad x = 0 \quad x = \pi \quad$ 4 intervals

d $y = \sqrt[3]{2x+5}$ $\quad x = -2 \quad x = 2 \quad$ 4 intervals

4 Use the trapezium rule with the stated number of equally-spaced ordinates to estimate the area of the region enclosed by the given curve, the x-axis and the given ordinates.

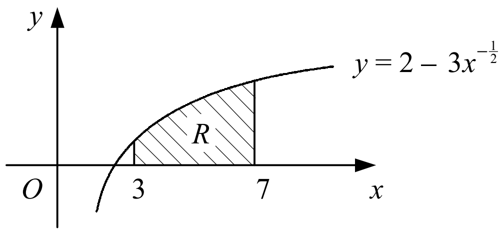
a $y = 3^x$ $\quad x = 0 \quad x = 3 \quad$ 4 ordinates

b $y = \sin(\lg x)$ $\quad x = 2 \quad x = 2.4 \quad$ 3 ordinates

c $y = \frac{x}{x^3+2}$ $\quad x = 0 \quad x = 0.5 \quad$ 6 ordinates

d $y = \sqrt{\cos(\frac{1}{2}x)}$ $\quad x = 0 \quad x = \frac{2\pi}{3} \quad$ 5 ordinates

5

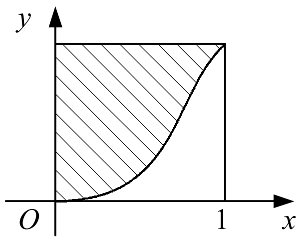


The diagram shows the finite region, R, which is bounded by the curve $y = 2 - 3x^{-\frac{1}{2}}$, the x-axis and the lines $x = 3$ and $x = 7$.

a Use the trapezium rule with five intervals to estimate the area of R.

b Use integration to find the exact area of R.

6



The diagram shows the curve $y = \sin x^2$, $0 \le x \le 1$ and the lines $x = 1$ and $y = \sin 1$.

a Use the trapezium rule with five strips of equal width to estimate the area bounded by the curve $y = \sin x^2$, the x-axis and the line $x = 1$, giving your answer to 4 decimal places.

The shaded region on the diagram is bounded by the curve, the y-axis and the line $y = \sin 1$. A flower bed is modelled by the shaded region, with the units on the axes in metres.

b Calculate an estimate for the area of the flower bed, correct to 2 significant figures.

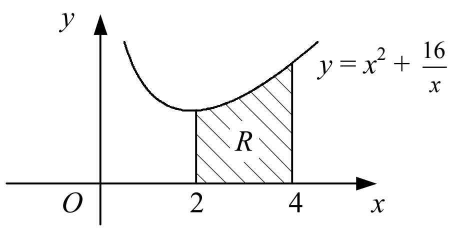
7 **a** Use the binomial theorem to expand $(1 + \frac{x}{2})^6$ in ascending powers of x up to and including the term in x^3.

The finite region R is bounded by the curve $y = (1 + \frac{x}{2})^6$, the coordinate axes and the line $x = 0.5$

b Use your expression in **a** and integration to find an estimate for the area of R.

c Use the trapezium rule with 6 equally-spaced ordinates to find another estimate for the area of R.

8



The diagram shows the curve $y = x^2 + \frac{16}{x}$ for $x > 0$.

a Show that the stationary point on the curve has coordinates (2, 12).

The region R is bounded by the curve $y = x^2 + \frac{16}{x}$, the x-axis and the lines $x = 2$ and $x = 4$.

b Use the trapezium rule with four intervals to estimate the area of R.

c State whether your answer to **b** is an under-estimate or an over-estimate of the area of R.