

- 1 Evaluate

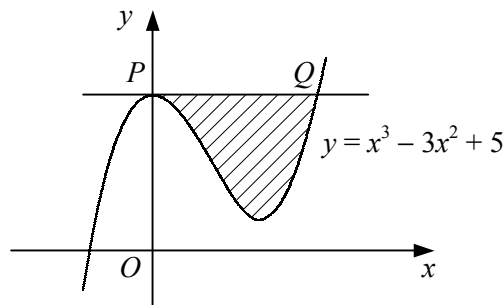
$$\int_2^3 \left(x^2 - \frac{3}{x^2}\right) dx.$$

- 2 The curve
- $y = f(x)$
- passes through the point
- $(1, -2)$
- and is such that

$$f'(x) = 1 - \frac{6}{x^3}.$$

Find the  $y$ -coordinate of the point on the curve where  $x = -1$ .

- 3



The diagram shows the curve with the equation  $y = x^3 - 3x^2 + 5$ .

The curve crosses the  $y$ -axis at the point  $P$ .

- a** Write down the coordinates of the point  $P$ .

The straight line passing through  $P$  parallel to the  $x$ -axis intersects the curve again at the point  $Q$ .

- b** Find the coordinates of the point  $Q$ .

- c** Find the area of the shaded region enclosed by the curve and the straight line  $PQ$ .

- 4 The curve
- $y = f(x)$
- passes through the point
- $(3, 22)$
- .

Given that

$$f'(x) = 3x^2 + 2x - 5,$$

- a** find an expression for  $f(x)$ .

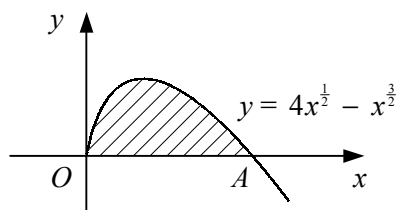
Given also that

$$g(x) = (x + 3)(x - 1)^2,$$

- b** show that  $g(x) = f(x) + 2$ ,

- c** sketch the curves  $y = f(x)$  and  $y = g(x)$  on the same set of axes.

- 5



The diagram shows the curve with the equation  $y = 4x^{1/2} - x^{3/2}$ .

The curve meets the  $x$ -axis at the origin,  $O$ , and at the point  $A$ .

- a** Find the coordinates of the point  $A$ .

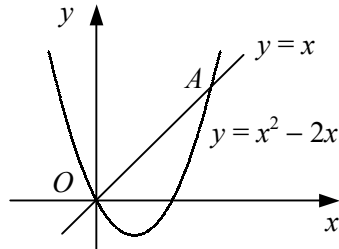
- b** Find the area of the shaded region enclosed by the curve and the  $x$ -axis.

6  $f(x) \equiv 3x^{\frac{1}{2}} - x^{-\frac{1}{2}}.$

a Find the value of  $f(2)$ , giving your answer in the form  $k\sqrt{2}$  where  $k$  is an exact fraction.

b Show that  $\int_3^4 f(x) \, dx = 12 - 4\sqrt{3}.$

7



The diagram shows the curve  $y = x^2 - 2x$  and the straight line  $y = x$ .

The curve and line intersect at the origin,  $O$ , and at the point  $A$ .

a Find the coordinates of the point  $A$ .

b Show that the area of the region enclosed by the curve and the line is  $\frac{9}{2}.$

8 Evaluate the improper integral

$$\int_0^8 \frac{3}{\sqrt{x}} \, dx,$$

giving your answer in the form  $k\sqrt{2}.$

9 Given that

$$\frac{dy}{dx} = 2x - \frac{3}{x^2}, \quad x \neq 0,$$

and that  $y = 0$  at  $x = 1$ ,

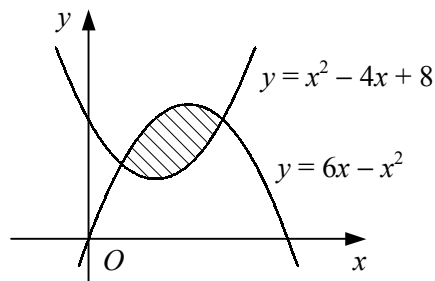
a find an expression for  $y$  in terms of  $x$ ,

b show that for all non-zero values of  $x$

$$x^2 \frac{d^2y}{dx^2} - 2y = k,$$

where  $k$  is a constant to be found.

10



The diagram shows the curves with the equations  $y = x^2 - 4x + 8$  and  $y = 6x - x^2$ .

a Find the coordinates of the points where the curves intersect.

b Show that the area of the shaded region enclosed by the two curves is given by

$$\int_1^4 (-8 + 10x - 2x^2) \, dx.$$

c Hence find the area of the shaded region.