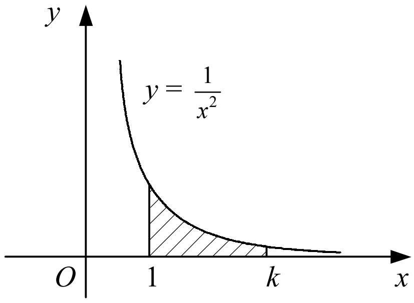
1



The diagram shows the curve $y = \frac{1}{x^2}$, $x > 0$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = k$, where $k > 1$.

a Find the area of the shaded region in terms of k.

b Hence evaluate

$$\int_1^{\infty} \frac{1}{x^2}\, dx.$$

2 Evaluate

a $\int_1^{\infty} \frac{1}{x^3}\, dx$ **b** $\int_1^{\infty} \frac{1}{x^{\frac{3}{2}}}\, dx$ **c** $\int_{-\infty}^{-1} \frac{3}{x^2}\, dx$ **d** $\int_2^{\infty} \frac{1}{x^3}\, dx$

e $\int_2^{\infty} \frac{9}{x^4}\, dx$ **f** $\int_4^{\infty} \frac{12}{x^{\frac{5}{2}}}\, dx$ **g** $\int_8^{\infty} \frac{4}{\sqrt[3]{x^4}}\, dx$ **h** $\int_{-\infty}^{-3} \frac{9}{2x^3}\, dx$

3 **a** Given that $0 < k < 1$, find in terms of k the value of the integral

$$\int_k^1 \frac{1}{x^{\frac{1}{2}}}\, dx.$$

b Hence evaluate

$$\int_0^1 \frac{1}{x^{\frac{1}{2}}}\, dx.$$

4 Evaluate

a $\int_0^1 \frac{1}{x^{\frac{1}{3}}}\, dx$ **b** $\int_0^1 \frac{1}{x^{\frac{2}{3}}}\, dx$ **c** $\int_0^9 \frac{2}{\sqrt{x}}\, dx$ **d** $\int_0^{16} \frac{1}{x^{\frac{1}{4}}}\, dx$

e $\int_{-1}^0 \frac{1}{\sqrt[5]{x}}\, dx$ **f** $\int_0^1 \frac{1}{2x^{\frac{3}{4}}}\, dx$ **g** $\int_{-8}^0 \frac{4}{x^{\frac{2}{3}}}\, dx$ **h** $\int_0^4 (3 + x^{-\frac{1}{2}})\, dx$

5 Given that

$$\int_3^{\infty} \frac{c}{x^2}\, dx = 2,$$

find the value of the constant c.

6 Given that $a > 0$ and that

$$\int_0^a \frac{4}{\sqrt[3]{x}}\, dx = 24,$$

find the value of the constant a.