C2 THE BINOMIAL THEOREM — Worksheet C

1 Expand $(1 + 4x)^4$ in ascending powers of x, simplifying the coefficients. **(4)**

2 Find the first three terms in the expansion of $(2 + 5x)^6$ in ascending powers of x, simplifying each coefficient. **(4)**

3 **a** Expand $(1 + 3x)^4$ in ascending powers of x, simplifying the coefficients. **(4)**

b Find the coefficient of x^2 in the expansion of

$$(1 + 4x - x^2)(1 + 3x)^4.$$ **(3)**

4 **a** Write down the first three terms in the binomial expansion of $(1 + ax)^n$, where n is a positive integer, in ascending powers of x. **(2)**

Given that the coefficient of x^2 is three times the coefficient of x,

b show that $n = \frac{6+a}{a}$. **(3)**

Given also that $a = \frac{2}{3}$,

c find the coefficient of x^3 in the expansion. **(3)**

5 **a** Expand $(1 + 3x)^7$ in ascending powers of x up to and including the term in x^4, simplifying each coefficient in the expansion. **(4)**

b Use your series with a suitable value of x to estimate the value of 1.03^7 correct to 5 decimal places. **(3)**

6 **a** Expand $(1 + x)^4$ in ascending powers of x. **(2)**

b Hence, or otherwise, write down the expansion of $(1 - x)^4$ in ascending powers of x. **(1)**

c By using your answers to parts **a** and **b**, or otherwise, solve the equation

$$(1 + x)^4 + (1 - x)^4 = 82,$$

for real values of x. **(5)**

7 Find the first four terms in the expansion of $(1 - 3x)^8$ in ascending powers of x, simplifying each coefficient. **(4)**

8 Given that

$$(1 + \tfrac{x}{2})^8(1 - x)^6 \equiv 1 + Ax + Bx^2 + \ldots,$$

find the values of the constants A and B. **(7)**

9 The first two terms in the expansion of $(1 + \frac{ax}{2})^{10} + (1 + bx)^{10}$, in ascending powers of x, are 2 and $90x^2$.

Given that $a < b$, find the values of the constants a and b. **(9)**

10 **a** Expand $(2 + x)^5$, simplifying the coefficient in each term. **(4)**

b Hence, or otherwise, write down the expansion of $(2 - x)^5$. **(1)**

c Show that

$$(2 + \sqrt{5})^5 - (2 - \sqrt{5})^5 = k\sqrt{5},$$

where k is an integer to be found. **(4)**

11 Expand $(1 - 2x)^5$ in ascending powers of x, simplifying each coefficient. **(4)**

12 **a** Find the first three terms in the expansion of $(1 + 4x)^7$ in ascending powers of x. **(3)**

b Hence, find the coefficient of x^2 in the expansion of

$$(1 + 2x)^2(1 + 4x)^7.$$ **(3)**

13 Given that

$$(k - x)^9 \equiv a - bx + bx^2 + \ldots,$$

find the values of the positive integers a, b and k. **(7)**

14 **a** Find the first four terms in the expansion of $(1 + 2x)^9$ in ascending powers of x. **(4)**

b Show that, if terms involving x^4 and higher powers of x may be ignored,

$$(1 + 2x)^9 + (1 - 2x)^9 = 2 + 288x^2.$$ **(3)**

c Hence find the value of

$$1.002^9 + 0.998^9,$$

giving your answer to 7 significant figures. **(2)**

15 Expand $(3 + 2x)^4$ in ascending powers of x, simplifying the coefficients. **(4)**

16 $$\mathrm{f}(x) \equiv (1 - x)(1 + 2x)^n, \quad n \in \mathbb{N}.$$

Given that the coefficient of x^2 in the binomial expansion of $\mathrm{f}(x)$ is 198, find

a the value of n, **(6)**

b the coefficient of x^3 in the expansion. **(3)**

17 **a** Expand $(1 - 2x)^4$ in ascending powers of x, simplifying the coefficients. **(3)**

b Hence, or otherwise, find the coefficient of y^2 in the expansion of

$$(1 + 4y - 2y^2)^4.$$ **(4)**

18 Expand $(\frac{3}{x} - x)^4$ in descending powers of x, simplifying the coefficient in each term. **(4)**

19 **a** Write down the first four terms in the expansion in ascending powers of x of

$$(1 + \frac{x}{k})^{2n},$$

where k is a non-zero constant, n is an integer and $n > 1$. **(4)**

Given that the coefficient of x^3 is half the coefficient of x^2,

b show that $3k = 4(n - 1)$. **(3)**

Given also that the coefficient of x is 2,

c find the values of n and k. **(3)**

20 The polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = (x + 3)^4 - (x + 1)^4.$$

a Show that $(x + 2)$ is a factor of $\mathrm{p}(x)$. **(2)**

b Fully factorise $\mathrm{p}(x)$. **(7)**

c Hence show that there is only one real solution to the equation $\mathrm{p}(x) = 0$. **(2)**