

1 Find the quotient obtained in dividing

a  $(x^3 + 2x^2 - x - 2)$  by  $(x + 1)$

b  $(x^3 + 2x^2 - 9x + 2)$  by  $(x - 2)$

c  $(20 + x + 3x^2 + x^3)$  by  $(x + 4)$

d  $(2x^3 - x^2 - 4x + 3)$  by  $(x - 1)$

e  $(x^3 - 2x + 21)$  by  $(x + 3)$

f  $(3x^3 + 16x^2 + 72)$  by  $(x + 6)$

g  $(2x^3 - 11x^2 - x + 3)$  by  $(2x - 1)$

h  $(4x^3 + 8x^2 + 7x + 30)$  by  $(2x + 5)$

2 Find the quotient and remainder obtained in dividing

a  $(x^3 + 8x^2 + 17x + 16)$  by  $(x + 5)$

b  $(x^3 - 15x^2 + 61x - 48)$  by  $(x - 7)$

c  $(3x^3 + 4x^2 + 7)$  by  $(2 + x)$

d  $(-x^3 - 5x^2 + 15x - 50)$  by  $(x + 8)$

e  $(4x^3 + 2x^2 - 16x + 3)$  by  $(2x - 5)$

f  $(1 - 22x^2 - 6x^3)$  by  $(3x + 2)$

3 Use the factor theorem to determine whether or not

a  $(x - 1)$  is a factor of  $(x^3 + 2x^2 - 2x - 1)$

b  $(x + 2)$  is a factor of  $(x^3 - 5x^2 - 9x + 2)$

c  $(x - 3)$  is a factor of  $(x^3 - x^2 - 14x + 27)$

d  $(x + 6)$  is a factor of  $(2x^3 + 13x^2 + 2x - 24)$

e  $(2x + 1)$  is a factor of  $(2x^3 - 5x^2 + 7x - 14)$

f  $(3x - 2)$  is a factor of  $(2 - 17x + 25x^2 - 6x^3)$

4  $f(x) \equiv x^3 - 2x^2 - 11x + 12.$

a Show that  $(x - 1)$  is a factor of  $f(x)$ .

b Hence, express  $f(x)$  as the product of three linear factors.

5  $g(x) \equiv 2x^3 + x^2 - 13x + 6.$

Show that  $(x + 3)$  is a factor of  $g(x)$  and solve the equation  $g(x) = 0$ .

6  $f(x) \equiv 6x^3 - 7x^2 - 71x + 12.$

Given that  $f(4) = 0$ , find all solutions to the equation  $f(x) = 0$ .

7  $g(x) \equiv x^3 + 7x^2 + 7x - 6.$

Given that  $x = -2$  is a solution to the equation  $g(x) = 0$ ,

a express  $g(x)$  as the product of a linear factor and a quadratic factor,

b find, to 2 decimal places, the other two solutions to the equation  $g(x) = 0$ .

8  $f(x) \equiv x^3 + 2x^2 - 11x - 12.$

a Evaluate  $f(1)$ ,  $f(2)$ ,  $f(-1)$  and  $f(-2)$ .

b Hence, state a linear factor of  $f(x)$  and fully factorise  $f(x)$ .

9 By first finding a linear factor, fully factorise

a  $x^3 - 2x^2 - 5x + 6$

b  $x^3 + x^2 - 5x - 2$

c  $20 + 11x - 8x^2 + x^3$

d  $3x^3 - 4x^2 - 35x + 12$

e  $x^3 + 8$

f  $12 + 29x + 8x^2 - 4x^3$

10 Solve each equation, giving your answers in exact form.

a  $x^3 - x^2 - 10x - 8 = 0$

b  $x^3 + 2x^2 - 9x - 18 = 0$

c  $4x^3 - 12x^2 + 9x = 2$

d  $x^3 - 5x^2 + 3x + 1 = 0$

e  $x^2(x + 4) = 3(3x + 2)$

f  $x^3 - 14x + 15 = 0$

- 11  $f(x) \equiv 2x^3 - x^2 - 15x + c.$   
Given that  $(x - 2)$  is a factor of  $f(x)$ ,  
**a** find the value of the constant  $c$ ,  
**b** fully factorise  $f(x)$ .
- 12  $g(x) \equiv x^3 + px^2 - 13x + q.$   
Given that  $(x + 1)$  and  $(x - 3)$  are factors of  $g(x)$ ,  
**a** show that  $p = 3$  and find the value of  $q$ ,  
**b** solve the equation  $g(x) = 0$ .
- 13 Use the remainder theorem to find the remainder obtained in dividing  
**a**  $(x^3 + 4x^2 - x + 6)$  by  $(x - 2)$                       **b**  $(x^3 - 2x^2 + 7x + 1)$  by  $(x + 1)$   
**c**  $(2x^3 + x^2 - 9x + 17)$  by  $(x + 5)$                       **d**  $(8x^3 + 4x^2 - 6x - 3)$  by  $(2x - 1)$   
**e**  $(2x^3 - 3x^2 - 20x - 7)$  by  $(2x + 1)$                       **f**  $(3x^3 - 6x^2 + 2x - 7)$  by  $(3x - 2)$
- 14 Given that when  $(x^3 - 4x^2 + 5x + c)$  is divided by  $(x - 2)$  the remainder is 5, find the value of the constant  $c$ .
- 15 Given that when  $(2x^3 - 9x^2 + kx + 5)$  is divided by  $(2x - 1)$  the remainder is  $-2$ , find the value of the constant  $k$ .
- 16 Given that when  $(2x^3 + ax^2 + 13)$  is divided by  $(x + 3)$  the remainder is 22,  
**a** find the value of the constant  $a$ ,  
**b** find the remainder when  $(2x^3 + ax^2 + 13)$  is divided by  $(x - 4)$ .
- 17  $f(x) \equiv px^3 + qx^2 + qx + 3.$   
Given that  $(x + 1)$  is a factor of  $f(x)$ ,  
**a** find the value of the constant  $p$ .  
Given also that when  $f(x)$  is divided by  $(x - 2)$  the remainder is 15,  
**b** find the value of the constant  $q$ .
- 18  $p(x) \equiv x^3 + ax^2 + 9x + b.$   
Given that  $(x - 3)$  is a factor of  $p(x)$ ,  
**a** find a linear relationship between the constants  $a$  and  $b$ .  
Given also that when  $p(x)$  is divided by  $(x + 2)$  the remainder is  $-30$ ,  
**b** find the values of the constants  $a$  and  $b$ .
- 19  $f(x) \equiv 4x^3 - 6x^2 + mx + n.$   
Given that when  $f(x)$  is divided by  $(x + 1)$  the remainder is 3 and that when  $f(x)$  is divided by  $(2x - 1)$  the remainder is 15, find the values of the constants  $m$  and  $n$ .
- 20  $g(x) \equiv x^3 + cx + 3.$   
Given that when  $g(x)$  is divided by  $(x - 4)$  the remainder is 39,  
**a** find the value of the constant  $c$ ,  
**b** find the quotient and remainder when  $g(x)$  is divided by  $(x + 2)$ .