1 **a** $a + d = 40$, $a + 4d = 121$

subtracting, $3d = 81$

$d = 27$

sub. $a = 13$

b $S_{25} = \frac{25}{2}[26 + (24 \times 27)] = 8425$

2 **a** $r = 3\sqrt{2} \div \sqrt{6} = \sqrt{3}$

$a = \sqrt{6} \div \sqrt{3} = \sqrt{2}$

b $S_8 = \dfrac{\sqrt{2}[(\sqrt{3})^8 - 1]}{\sqrt{3} - 1}$

$= \dfrac{80\sqrt{2}}{\sqrt{3} - 1} \times \dfrac{\sqrt{3} + 1}{\sqrt{3} + 1}$

$= \dfrac{80\sqrt{2}(\sqrt{3} + 1)}{3 - 1}$

$= 40\sqrt{2}(\sqrt{3} + 1)$

3 **a** 3, 7, 11, 15, 19

b AP: $a = 3$, $d = 4$, $n = 20$

$S_{20} = \frac{20}{2}[6 + (19 \times 4)]$

$= 820$

4 **a** $= 12000 \times (0.75)^4$

$= 3796.875$

$= £3800$ (3sf)

b GP: $a = 12000$, $r = 0.75$

$S_8 = \dfrac{12000[1 - (0.75)^8]}{1 - 0.75}$

$= £43\,200$ (3sf)

5 $t_2 = 4 + 2k$

$t_3 = 4 - k(4 + 2k)$

$\therefore\ 4 - 4k - 2k^2 = 3$

$2k^2 + 4k - 1 = 0$

$k = \dfrac{-4 \pm \sqrt{16 + 8}}{4} = \dfrac{-4 \pm 2\sqrt{6}}{4}$

$k > 0 \ \therefore\ k = -1 + \frac{1}{2}\sqrt{6}$

6 $\sum_{r=1}^{9} 3^r$: GP, $a = 3$, $r = 3$

$S_9 = \dfrac{3(3^9 - 1)}{3 - 1} = 29523$

$\therefore\ \sum_{r=1}^{9} (3^r - 1) = 29523 - 9$

$= 29\,514$

7 **a** 2 years = 8×3 months

total $= 3 \times S_8$ [AP: $a = 40$, $d = 2$]

$= 3 \times \frac{8}{2}[80 + (7 \times 2)]$

$= 3 \times 376 = £1128$

b n years = $4n \times 3$ months

total $= 3 \times S_{4n}$

$= 3 \times \frac{4n}{2}\{80 + [(4n - 1) \times 2]\}$

$= 6n(80 + 8n - 2)$

$= 12n(4n + 39)$

8 **a** $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$

$rS_n = ar + ar^2 + \ldots + ar^{n-1} + ar^n$

subtracting, $S_n - rS_n = a - ar^n$

$S_n(1 - r) = a(1 - r^n)$

$S_n = \dfrac{a(1 - r^n)}{1 - r}$

b $r = 6 \div 3 = 2$

$a \times 2^3 = 3 \ \therefore\ a = \frac{3}{8}$

$S_{16} = \dfrac{\frac{3}{8}(2^{16} - 1)}{2 - 1}$

$= 24\,575\frac{5}{8}$

9 **a** AP: $a = 3, d = 3$

$500 \div 3 = 166\frac{2}{3}$ $\therefore n = 166$

$S_{166} = \frac{166}{2}[6 + (165 \times 3)]$

$= 41\,583$

b GP: $a = 8, r = 2, n = 10$

$S_{10} = \frac{8(2^{10} - 1)}{2 - 1}$

$= 8184$

10 **a** $(4k - 2) - (k + 4) = (k^2 - 2) - (4k - 2)$

$3k - 6 = k^2 - 4k$

$k^2 - 7k + 6 = 0$

b $(k - 1)(k - 6) = 0$, $k = 1$ or 6

$d = 3k - 6$

$d > 0$ $\therefore k = 6, a = 10, d = 12$

$u_{15} = 10 + (14 \times 12) = 178$

11 **a** new subscribers in 4th week

$= 200 \times (1.15)^3 = 304.175$

$= 304$ (nearest unit)

b new subscribers: GP, $a = 200, r = 1.15$

$S_{10} = \frac{200[(1.15)^{10} - 1]}{1.15 - 1} = 4060.74$

total no. of subscribers $= 3600 + S_{10}$

$= 7661$ (nearest unit)

12 **a** **i** $= 3 \times 570 = 1710$

ii $= 570 + (2 \times 30) = 630$

iii $= 570 + (\frac{1}{2} \times 30 \times 31) = 1035$

b $S_{30} = \frac{30}{2}[-20 + 29d] = 570$

$d = 2$

$\therefore t_2 = -10 + 2 = -8$

13 **a** $a + 2d = 298$, $a + 7d = 263$

subtracting, $5d = -35$

$d = -7$

b sub. $a = 312$

$312 - 7(n - 1) > 0$

$n < \frac{319}{7}$ $\therefore$ 45 positive terms

c max S_n when $n = 45$

$S_{45} = \frac{45}{2}[624 + (44 \times -7)] = 7110$

14 **a** AP: $a = 10, d = 6$

$S_n = \frac{n}{2}[20 + 6(n - 1)]$

$= n(3n + 7)$

b $S_{2n} = 2n[(3 \times 2n) + 7]$

$= 12n^2 + 14n$

required sum $= S_{2n} - S_n$

$= (12n^2 + 14n) - (3n^2 + 7n)$

$= 9n^2 + 7n = n(9n + 7)$

15 **a** $\frac{162}{1 - r} = 486$

$1 - r = \frac{162}{486} = \frac{1}{3}$ $\therefore r = \frac{2}{3}$

b $u_6 = 162 \times (\frac{2}{3})^5 = \frac{64}{3}$ or $21\frac{1}{3}$

c $S_{10} = \frac{162[1 - (\frac{2}{3})^{10}]}{1 - \frac{2}{3}} = 477.572$

16 **a** $3(x - 3) = y - 3$

$y = 3x - 6$

b $\left(\frac{x}{3}\right)^3 = \frac{y}{3}$

$x^3 = 9y = 9(3x - 6)$

$x^3 - 27x + 54 = 0$

c trying $x = 1, 2$ etc. $\Rightarrow$ $x = 3$ is a solution

$\therefore$ $(x - 3)$ is a factor

$$
\begin{array}{r|l}
 & x^2 + 3x - 18 \\
x - 3 & x^3 + 0x^2 - 27x + 54 \\
 & x^3 - 3x^2 \\
 & 3x^2 - 27x \\
 & 3x^2 - 9x \\
 & -18x + 54 \\
 & -18x + 54 \\
\end{array}
$$

$(x - 3)(x^2 + 3x - 18) = 0$

$(x - 3)(x + 6)(x - 3) = 0$

$x = -6$ or 3