

- 1 a** $a + 2d = -10$ (1)
 $\frac{8}{2}(2a + 7d) = 16 \Rightarrow 2a + 7d = 4$
 $2 \times (1) \Rightarrow 2a + 4d = -20$
 subtracting, $3d = 24$
 $d = 8$
 sub. $a = -26$
- b** $-26 + 8(n - 1) > 300$
 $n > 41\frac{3}{4} \therefore$ smallest $n = 42$
- 3 a** $\frac{9}{2}(2a + 8d) = 126$
 $9(a + 4d) = 126$
 $a + 4d = 14$
- b** $\frac{15}{2}(2a + 14d) = 277.5$
 $a + 7d = 18.5$
 subtracting, $3d = 4.5$
 $d = 1.5$
 sub. $a = 8$
- c** $S_{32} = \frac{32}{2} [16 + (31 \times 1.5)] = 1000$
- 5 a** AP: $a = 4, l = 120, n = 30$
 $S_{30} = \frac{30}{2} (4 + 120) = 1860$
- b i** $= \sum_{r=1}^{30} 4r + 30 = 1890$
- ii** $= 2 \times \sum_{r=1}^{30} 4r - (30 \times 5)$
 $= (2 \times 1860) - 150 = 3570$
- 7 a** $S_n = 2 + 4 + 6 + \dots + (2n - 2) + 2n$
 write in reverse
 $S_n = 2n + (2n - 2) + \dots + 6 + 4 + 2$
 adding, $2S_n = n \times (2n + 2)$
 $S_n = n(n + 1)$
- b** integers 200 to 800, AP: $n = 601$
 $S_{601} = \frac{601}{2} (200 + 800) = 300\,500$
 integers 200 to 800 divisible by 4
 AP: $a = 200, l = 800$
 $200 + 4(n - 1) = 800 \Rightarrow n = 151$
 $S_{151} = \frac{151}{2} (200 + 800) = 75\,500$
 required sum = $300\,500 - 75\,500$
 $= 225\,000$
- 2 a** $a + 2d = \frac{5}{6}$
 $a + 6d = 2\frac{1}{3}$
 subtracting, $4d = 1\frac{1}{2}$
 $d = \frac{3}{8}$
 sub. $a = \frac{1}{12}$
- b** $S_n = \frac{n}{2} [\frac{1}{6} + \frac{3}{8}(n - 1)]$
 $= \frac{1}{48} n[4 + 9(n - 1)]$
 $= \frac{1}{48} n(9n - 5) \quad [k = \frac{1}{48}]$
- 4 a** $(5k + 3) - (7k - 1) = (4k + 1) - (5k + 3)$
 $-2k + 4 = -k - 2$
 $k = 6$
- b** given terms = 41, 33, 25
 $d = -8$
 smallest +ve term = $25 + (3 \times -8) = 1$
- c** consider series of +ve terms in reverse
 $a = 1, d = 8$
 $S_r = \frac{r}{2} [2 + 8(r - 1)] = r(4r - 3)$
- 6 a** $500 + (7 \times 40) = \text{£}780$
- b** AP: $a = 500, d = 40$
 $S_n = \frac{n}{2} [1000 + 40(n - 1)] = 20n(n + 24)$
- c** AP: $a = 400, d = 60$
 $S_n = \frac{n}{2} [800 + 60(n - 1)] = 10n(3n + 37)$
 $\therefore 20n(n + 24) = 10n(3n + 37)$
 $n \neq 0 \therefore 2(n + 24) = (3n + 37)$
 $n = 11 \therefore 11$ years
- 8 a** $S_n = \frac{1}{2} n[2a + (n - 1)d]$
- b** $S_2 = \frac{2}{2} (2a + d) = 2a + d$
 $S_6 = \frac{6}{2} (2a + 5d) = 6a + 15d$
 $S_8 = \frac{8}{2} (2a + 7d) = 8a + 28d$
 $2(S_6 - S_2) = 2[(6a + 15d) - (2a + d)]$
 $= 2(4a + 14d)$
 $= 8a + 28d = S_8$
- c** for +ve terms $40 - 3(n - 1) > 0$
 $n < \frac{43}{3} \therefore 14$ terms
 $\therefore S_{14} = \frac{14}{2} [80 + (13 \times -3)] = 287$

9 a i $u_4 - u_1 = x + 3$

$$u_7 = u_4 + (x + 3) = 3x + 6$$

ii $3d = x + 3$

$$d = \frac{1}{3}x + 1$$

iii $S_{10} = \frac{10}{2} [2x + 9(\frac{1}{3}x + 1)]$
 $= 5[2x + 3x + 9] = 25x + 45$

b $x + 19(\frac{1}{3}x + 1) = 52$

$$3x + 19x + 57 = 156$$

$$x = \frac{99}{22} = \frac{9}{2} \text{ or } 4\frac{1}{2}$$

10 $S_{20} = \frac{20}{2} (2a + 19d) = 20a + 190d$

$$S_{30} = \frac{30}{2} (2a + 29d) = 30a + 435d$$

$$S_{30} - S_{20} = 10a + 245d$$

$$\therefore 20a + 190d = 10a + 245d$$

$$10a = 55d$$

$$2a = 11d$$

$$\therefore a : d = 11 : 2$$

11 a $S_6 = 12(16 - 6) = 120$

$$S_5 = 10(16 - 5) = 110$$

$$u_6 = S_6 - S_5 = 10$$

b $S_n = 2n(16 - n) = 32n - 2n^2$

$$S_{n-1} = 2(n-1)[16 - (n-1)]$$

$$= 2(n-1)(17 - n)$$

$$= -2n^2 + 36n - 34$$

$$u_n = S_n - S_{n-1}$$

$$= (32n - 2n^2) - (-2n^2 + 36n - 34)$$

$$= 34 - 4n$$

c $u_{n-1} = 34 - 4(n-1) = 38 - 4n$

$$u_n - u_{n-1} = (34 - 4n) - (38 - 4n) = -4$$

$$u_n - u_{n-1} \text{ constant } \therefore \text{ arithmetic series}$$

12 a i $2400 + (5 \times 250) = 3650$

ii AP: $a = 2400, d = 250$

$$S_{10} = \frac{10}{2} [4800 + (9 \times 250)]$$

$$= 35\,250$$

b AP: $a = 2400, d = C$

$$\frac{10}{2} [4800 + (9 \times C)] = 40\,000$$

$$C = \frac{3200}{9} = 356 \text{ (nearest unit)}$$

13 a let common difference be d

$$S_r = a + (a + d) + (a + 2d) + \dots + (l - 2d) + (l - d) + l$$

write in reverse

$$S_r = l + (l - d) + (l - 2d) + \dots + (a + 2d) + (a + d) + a$$

adding, $2S_r = r \times (a + l)$

$$S_r = \frac{1}{2} r(a + l)$$

b $n = 18, l = 68, S_{18} = 153$

$$\therefore 153 = \frac{18}{2} (a + 68)$$

$$a = 17 - 68 = -51$$