1 **a** $= [-2x^{-1}]_1^4$
$= -\frac{1}{2} - (-2)$
$= \frac{3}{2}$

b $= \int_0^2 (x^2 - 6x + 9)\ dx$
$= [\frac{1}{3}x^3 - 3x^2 + 9x]_0^2$
$= (\frac{8}{3} - 12 + 18) - 0$
$= 8\frac{2}{3}$

2 **a** $2x^2 - 6x + 5 = x^2 - 3x + 5$
$x^2 - 3x = 0$
$x(x - 3) = 0$
$x = 0, 3$
$\therefore$ (0, 5) and (3, 5)

b area $= \int_0^3 [(x^2 - 3x + 5) - (2x^2 - 6x + 5)]\ dx$
$= \int_0^3 (3x - x^2)\ dx$

c $= [\frac{3}{2}x^2 - \frac{1}{3}x^3]_0^3$
$= (\frac{27}{2} - 9) - 0$
$= \frac{9}{2}$

3 $= \int \frac{x^2 - 2x + 1}{x^{\frac{1}{2}}}\ dx$
$= \int (x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x^{-\frac{1}{2}})\ dx$
$= \frac{2}{5}x^{\frac{5}{2}} - \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$

4 **a** $y = \int (k - x^{-\frac{1}{2}})\ dx$
$y = kx - 2x^{\frac{1}{2}} + c$
$(1, -2) \Rightarrow -2 = k - 2 + c$
$0 = k + c \quad (1)$
$(4, 5) \Rightarrow 5 = 4k - 4 + c$
$9 = 4k + c \quad (2)$
$(2) - (1) \quad 9 = 3k$
$k = 3$

b $c = -3$
$\therefore\ y = 3x - 2x^{\frac{1}{2}} - 3$
when $y = 2$
$3x - 2x^{\frac{1}{2}} - 3 = 2$
$3x - 2x^{\frac{1}{2}} - 5 = 0$
$(3x^{\frac{1}{2}} - 5)(x^{\frac{1}{2}} + 1) = 0$
$x^{\frac{1}{2}} = -1$ [no sols] or $\frac{5}{3}$
$\therefore\ x = \frac{25}{9}$

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5 **a** at A, $x = 0 \Rightarrow (0, 2)$

$\frac{dy}{dx} = -1 - 2x$

grad at $A = -1$

$\therefore\ y = 2 - x$

b curve cuts x-axis when $y = 0$

$2 - x - x^2 = 0$

$(2 + x)(1 - x) = 0$

$x = -2, 1$

area below curve

$= \int_0^1 (2 - x - x^2)\ dx$

$= [2x - \frac{1}{2}x^2 - \frac{1}{3}x^3]_0^1$

$= (2 - \frac{1}{2} - \frac{1}{3}) - 0 = \frac{7}{6}$

tangent cuts x-axis when $y = 0 \Rightarrow x = 2$

area below line $= \frac{1}{2} \times 2 \times 2 = 2$

shaded area $= 2 - \frac{7}{6} = \frac{5}{6}$

6 **a**

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.319	1.024	0

b $\approx \frac{1}{2} \times \frac{\pi}{6} \times [1 + 0 + 2(1.319 + 1.024)]$

$= 1.49$ (3sf)

c under-estimate

curve passes above top of each trapezium

7 $\int_4^\infty 5x^{-\frac{3}{2}}\ dx$

$= \lim_{k\to\infty} [-10x^{-\frac{1}{2}}]_4^k$

$= \lim_{k\to\infty} [(-10k^{-\frac{1}{2}}) - (-5)]$

$= \lim_{k\to\infty} (5 - 10k^{-\frac{1}{2}})$

$= 5$

8 **a** $f(x) = \int (4x^3 - 8x)\ dx$

$= x^4 - 4x^2 + c$

$(2, -5) \Rightarrow -5 = 16 - 16 + c$

$c = -5$

$\therefore\ f(x) = x^4 - 4x^2 - 5$

b $x^4 - 4x^2 - 5 = 0$

$(x^2 + 1)(x^2 - 5) = 0$

$x^2 = -1$ [no sols] or 5

$x = \pm\sqrt{5}$

$\therefore\ (-\sqrt{5}, 0), (\sqrt{5}, 0)$

9 **a** at A, $x = 0 \ \therefore\ A\ (0, 4)$

at B, $y = 0$

$(x^{\frac{1}{2}} - 2)^2 = 0$

$x^{\frac{1}{2}} = 2$

$x = 4 \ \therefore\ B\ (4, 0)$

b $= \int_0^4 (x - 4x^{\frac{1}{2}} + 4)\ dx$

$= [\frac{1}{2}x^2 - \frac{8}{3}x^{\frac{3}{2}} + 4x]_0^4$

$= (8 - \frac{64}{3} + 16) - 0$

$= \frac{8}{3}$

10 **a** (2, 0)

b

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$(2-x)^3$	8	$\frac{27}{8}$	1	$\frac{1}{8}$	0

area $\approx \frac{1}{2} \times \frac{1}{2} \times [8 + 0 + 2(\frac{27}{8} + 1 + \frac{1}{8})]$

$= 4\frac{1}{4}$

c $= 2^3 + 3(2^2)(-x) + 3(2)(-x)^2 + (-x)^3$

$= 8 - 12x + 6x^2 - x^3$

d area $= \int_0^2 (8 - 12x + 6x^2 - x^3)\ dx$

$= [8x - 6x^2 + 2x^3 - \frac{1}{4}x^4]_0^2$

$= (16 - 24 + 16 - 4) - 0 = 4$

$\therefore$ % error $= \frac{4\frac{1}{4} - 4}{4} \times 100\% = 6.25\%$

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