1 $= 1 + 4(4x) + 6(4x)^2 + 4(4x)^3 + (4x)^4$
$= 1 + 16x + 96x^2 + 256x^3 + 256x^4$

2 $= 2^6 + 6(2^5)(5x) + \frac{6\times5}{2}(2^4)(5x)^2 + \ldots$
$= 64 + 960x + 6000x^2 + \ldots$

3 **a** $= 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$
$= 1 + 12x + 54x^2 + 108x^3 + 81x^4$

b term in $x^2 = (1)(54x^2) + (4x)(12x) + (-x^2)(1)$
coefficient of $x^2 = 54 + 48 - 1 = 101$

4 **a** $= 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$
$= 1 + anx + \frac{1}{2}a^2n(n-1)x^2 + \ldots$

b $\frac{1}{2}a^2n(n-1) = 3an$
$a^2n(n-1) = 6an$
$an[a(n-1) - 6] = 0$
$n \neq 0 \;\therefore\; a(n-1) - 6 = 0$
$an - a = 6$
$n = \frac{6+a}{a}$

c $n = 10$
$\therefore$ coefficient of $x^3 = \frac{10\times9\times8}{3\times2} \times (\frac{2}{3})^3 = \frac{320}{9}$ or $35\frac{5}{9}$

5 **a** $= 1 + 7(3x) + \frac{7\times6}{2}(3x)^2 + \frac{7\times6\times5}{3\times2}(3x)^3 + \frac{7\times6\times5\times4}{4\times3\times2}(3x)^4 + \ldots$
$= 1 + 21x + 189x^2 + 945x^3 + 2835x^4 + \ldots$

b let $x = 0.01$
$1.03^7 \approx 1 + 0.21 + 0.0189 + 0.000\,945 + 0.000\,028\,35$
$= 1.229\,87$ (5dp)

6 **a** $= 1 + 4x + 6x^2 + 4x^3 + x^4$

b $= 1 - 4x + 6x^2 - 4x^3 + x^4$

c $1 + 4x + 6x^2 + 4x^3 + x^4 + 1 - 4x + 6x^2 - 4x^3 + x^4 = 82$
$2 + 12x^2 + 2x^4 = 82$
$x^4 + 6x^2 - 40 = 0$
$(x^2 + 10)(x^2 - 4) = 0$
$x^2 = -10$ [no real solutions]
or $x^2 = 4$
$x = \pm 2$

7 $= 1 + 8(-3x) + \frac{8\times7}{2}(-3x)^2 + \frac{8\times7\times6}{3\times2}(-3x)^3 + \ldots$
$= 1 - 24x + 252x^2 - 1512x^3 + \ldots$

8 $= [1 + 8(\frac{x}{2}) + \frac{8\times7}{2}(\frac{x}{2})^2 + \ldots][1 + 6(-x) + \frac{6\times5}{2}(-x)^2 + \ldots]$
$= [1 + 4x + 7x^2 + \ldots][1 - 6x + 15x^2 + \ldots]$
$= 1 - 6x + 15x^2 + 4x - 24x^2 + 7x^2 + \ldots$
$= 1 - 2x - 2x^2 + \ldots$
$\therefore A = -2, B = -2$

9 $(1 + \frac{ax}{2})^{10} + (1 + bx)^{10} = 1 + 10(\frac{ax}{2}) + \frac{10\times9}{2}(\frac{ax}{2})^2 + \ldots + 1 + 10(bx) + \frac{10\times9}{2}(bx)^2 + \ldots$

$= 2 + (5a + 10b)x + (\frac{45}{4}a^2 + 45b^2)x^2 + \ldots$

$\therefore \quad 5a + 10b = 0 \quad \Rightarrow \quad a = -2b$

and $\quad \frac{45}{4}a^2 + 45b^2 = 90 \quad \Rightarrow \quad a^2 + 4b^2 = 8$

sub. $(-2b)^2 + 4b^2 = 8$

$b^2 = 1$

$a < b \ \therefore\ b = 1, a = -2$

10 **a** $= 2^5 + 5(2^4)x + 10(2^3)x^2 + 10(2^2)x^3 + 5(2)x^4 + x^5$

$= 32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$

b $= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$

c $(2 + \sqrt{5})^5 = 32 + 80(\sqrt{5}) + 80(\sqrt{5})^2 + 40(\sqrt{5})^3 + 10(\sqrt{5})^4 + (\sqrt{5})^5$

$= 32 + 80\sqrt{5} + 400 + 200\sqrt{5} + 250 + 25\sqrt{5}$

$= 682 + 305\sqrt{5}$

$\therefore (2 + \sqrt{5})^5 - (2 - \sqrt{5})^5 = (682 + 305\sqrt{5}) - (682 - 305\sqrt{5})$

$= 610\sqrt{5},\ k = 610$

11 $= 1 + 5(-2x) + 10(-2x)^2 + 10(-2x)^3 + 5(-2x)^4 + (-2x)^5$

$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

12 **a** $= 1 + 7(4x) + \frac{7\times6}{2}(4x)^2 + \ldots$

$= 1 + 28x + 336x^2 + \ldots$

b $(1 + 2x)^2(1 + 4x)^7 = (1 + 4x + 4x^2)(1 + 28x + 336x^2 + \ldots)$

term in $x^2 = (1)(336x^2) + (4x)(28x) + (4x^2)(1)$

coefficient of $x^2 = 336 + 112 + 4 = 452$

13 $(k - x)^9 = k^9 + 9(k^8)(-x) + \frac{9\times8}{2}(k^7)(-x)^2 + \ldots$

$= k^9 - 9k^8x + 36k^7x^2 + \ldots$

$\therefore\ -b = -9k^8$ and $b = 36k^7$

$9k^8 = 36k^7$

$9k^7(k - 4) = 0$

$k \neq 0\ \therefore\ k = 4$

$a = k^9 = 262\,144$

$b = 9k^8 = 589\,824$

14 **a** $= 1 + 9(2x) + \frac{9\times8}{2}(2x)^2 + \frac{9\times8\times7}{3\times2}(2x)^3 + \ldots$

$= 1 + 18x + 144x^2 + 672x^3 + \ldots$

b $(1 - 2x)^9 = 1 - 18x + 144x^2 - 672x^3 + \ldots$

$\therefore\ (1 + 2x)^9 + (1 - 2x)^9 = (1 + 18x + 144x^2 + 672x^3 + \ldots) + (1 - 18x + 144x^2 - 672x^3 + \ldots)$

$= 2 + 288x^2$ (ignoring terms in x^4 and higher powers of x)

c $1.002^9 + 0.998^9 = (1 + 2x)^9 + (1 - 2x)^9$ with $x = 0.001$

$\therefore\ 1.002^9 + 0.998^9 \approx 2 + 0.000\,288$

$= 2.000\,288$ (7sf)

15 $= 3^4 + 4(3)^3(2x) + 6(3)^2(2x)^2 + 4(3)(2x)^3 + (2x)^4$

$= 81 + 216x + 216x^2 + 96x^3 + 16x^4$

16 **a** $(1-x)(1+2x)^n = (1-x)[1 + n(2x) + \frac{n(n-1)}{2}(2x)^2 + \ldots]$

$= (1-x)[1 + 2nx + 2n(n-1)x^2 + \ldots]$

$\therefore\ 2n(n-1) - 2n = 198$

$n^2 - 2n - 99 = 0$

$(n+9)(n-11) = 0$

$n \geq 0\ \ \therefore\ n = 11$

b $(1-x)(1+2x)^{11} = (1-x)[\ldots + \frac{11\times10}{2}(2x)^2 + \frac{11\times10\times9}{3\times2}(2x)^3 + \ldots]$

$= (1-x)[\ldots + 220x^2 + 1320x^3 + \ldots]$

$\therefore$ coefficient of $x^3 = 1320 - 220 = 1100$

17 **a** $= 1 + 4(-2x) + 6(-2x)^2 + 4(-2x)^3 + (-2x)^4$

$= 1 - 8x + 24x^2 - 32x^3 + 16x^4$

b let $x = y^2 - 2y$

$(1 + 4y - 2y^2)^4 = 1 - 8(y^2 - 2y) + 24(y^2 - 2y)^2 + \ldots$

term in $y^2 = -8y^2 + 24(-2y)^2$

coefficient of $y^2 = -8 + 96 = 88$

18 $= (\frac{3}{x})^4 + 4(\frac{3}{x})^3(-x) + 6(\frac{3}{x})^2(-x)^2 + 4(\frac{3}{x})(-x)^3 + (-x)^4$

$= x^4 - 12x^2 + 54 - \frac{108}{x^2} + \frac{81}{x^4}$

19 **a** $= 1 + 2n(\frac{x}{k}) + \frac{2n(2n-1)}{2}(\frac{x}{k})^2 + \frac{2n(2n-1)(2n-2)}{3\times2}(\frac{x}{k})^3 + \ldots$

$= 1 + \frac{2n}{k}x + \frac{n(2n-1)}{k^2}x^2 + \frac{2n(n-1)(2n-1)}{3k^3}x^3 + \ldots$

b $\frac{2n(n-1)(2n-1)}{3k^3} = \frac{1}{2} \times \frac{n(2n-1)}{k^2}$

$4n(n-1)(2n-1) = 3kn(2n-1)$

$n(2n-1)[4(n-1) - 3k] = 0$

$n > 1\ \ \therefore\ 4(n-1) - 3k = 0$

$3k = 4(n-1)$

c $\frac{2n}{k} = 2\ \ \therefore\ n = k$

$\therefore\ 3k = 4k - 4$

$k = 4,\ \ n = 4$

20 **a** $p(-2) = 1^4 - (-1)^4 = 1 - 1 = 0$

$\therefore\ (x+2)$ is a factor of $p(x)$

b $p(x) = [x^4 + 4(x^3)(3) + 6(x^2)(3^2) + 4(x)(3^3) + 3^4]$

$- [x^4 + 4x^3 + 6x^2 + 4x + 1]$

$= 8x^3 + 48x^2 + 104x + 80$

$= 8(x^3 + 6x^2 + 13x + 10)$

$$
\begin{array}{r|l}
 & x^2 + 4x + 5 \\
x+2 & x^3 + 6x^2 + 13x + 10 \\
 & x^3 + 2x^2 \\
 & 4x^2 + 13x \\
 & 4x^2 + 8x \\
 & 5x + 10 \\
 & 5x + 10
\end{array}
$$

$p(x) = 8(x+2)(x^2 + 4x + 5)$

c $8(x+2)(x^2 + 4x + 5) = 0$

$x = -2$ or $(x^2 + 4x + 5) = 0$

$b^2 - 4ac = 16 - 20 = -4$

$b^2 - 4ac < 0\ \ \therefore$ no real solutions to $(x^2 + 4x + 5) = 0$

$\therefore$ only one real solution to $p(x) = 0$