**1** **a** $f(-2) = -8 + 4 + 44 - 40 = 0$

$\therefore\ (x + 2)$ is a factor of $f(x)$

**b**

$$
\begin{array}{r|rrrr}
 & x^2 & -\ x & -\ 20 & \\ \hline
x+2 & x^3 & +\ x^2 & -22x & -40 \\
 & x^3 & +2x^2 & & \\ \cline{2-3}
 & & -\ x^2 & -22x & \\
 & & -\ x^2 & -\ 2x & \\ \cline{3-4}
 & & & -20x & -40 \\
 & & & -20x & -40 \\ \cline{4-5}
\end{array}
$$

$\therefore\ f(x) \equiv (x + 2)(x^2 - x - 20)$

$\equiv (x + 2)(x + 4)(x - 5)$

**c** $f(x) = 0 \Rightarrow (x + 2)(x + 4)(x - 5) = 0$

$x = -4, -2$ or $5$

**2** **a** $f(2) = f(-3)$

$\therefore\ 8 - 8 + 2k + 1 = -27 - 18 - 3k + 1$

$k = -9$

**b** $= f(-2) = -8 - 8 + 18 + 1 = 3$

**3** **a** $= p(-2) = -16 - 36 + 4 + 11 = -37$

**b**

$$
\begin{array}{r|rrrr}
 & x^2 & -\ 4x & -\ 3 & \\ \hline
2x-1 & 2x^3 & -\ 9x^2 & -\ 2x & +11 \\
 & 2x^3 & -\ x^2 & & \\ \cline{2-3}
 & & -\ 8x^2 & -\ 2x & \\
 & & -\ 8x^2 & +\ 4x & \\ \cline{3-4}
 & & & -\ 6x & +11 \\
 & & & -\ 6x & +\ 3 \\ \cline{4-5}
 & & & & 8
\end{array}
$$

$\therefore$ quotient $= x^2 - 4x - 3$

remainder $= 8$

**4** **a** $A$ is $(0, 12)$

**b** $x = 1$ is a root of $y = 0$

$\therefore\ (x - 1)$ is a factor of $y$

$$
\begin{array}{r|rrrr}
 & x^2 & -\ 4x & -\ 12 & \\ \hline
x-1 & x^3 & -\ 5x^2 & -\ 8x & +12 \\
 & x^3 & -\ x^2 & & \\ \cline{2-3}
 & & -\ 4x^2 & -\ 8x & \\
 & & -\ 4x^2 & +\ 4x & \\ \cline{3-4}
 & & & -12x & +12 \\
 & & & -12x & +12 \\ \cline{4-5}
\end{array}
$$

$\therefore\ y = (x - 1)(x^2 - 4x - 12)$

$= (x - 1)(x + 2)(x - 6)$

$\therefore\ y = 0$ when $x = -2, 1$ or $6$

$\therefore\ B$ is $(-2, 0)$ and $D$ is $(6, 0)$

**5** **a** $f(1) = 0$

$\therefore\ 1 - 3 + k + 8 = 0$

$k = -6$

**b**

$$
\begin{array}{r|rrrr}
 & x^2 & -\ 2x & -\ 8 & \\ \hline
x-1 & x^3 & -\ 3x^2 & -\ 6x & +\ 8 \\
 & x^3 & -\ x^2 & & \\ \cline{2-3}
 & & -\ 2x^2 & -\ 6x & \\
 & & -\ 2x^2 & +\ 2x & \\ \cline{3-4}
 & & & -\ 8x & +\ 8 \\
 & & & -\ 8x & +\ 8 \\ \cline{4-5}
\end{array}
$$

$\therefore\ f(x) = (x - 1)(x^2 - 2x - 8)$

$= (x - 1)(x + 2)(x - 4)$

$f(x) = 0 \Rightarrow x = -2, 1, 4$

**6** let $f(x) = 2x^3 + x^2 - 13x + 6$

$f(1) = -4,\ f(2) = 0$

$\therefore\ (x - 2)$ is a factor of $f(x)$

$$
\begin{array}{r|rrrr}
 & 2x^2 & +\ 5x & -\ 3 & \\ \hline
x-2 & 2x^3 & +\ x^2 & -13x & +\ 6 \\
 & 2x^3 & -\ 4x^2 & & \\ \cline{2-3}
 & & 5x^2 & -13x & \\
 & & 5x^2 & -10x & \\ \cline{3-4}
 & & & -\ 3x & +\ 6 \\
 & & & -\ 3x & +\ 6 \\ \cline{4-5}
\end{array}
$$

$\therefore\ (x - 2)(2x^2 + 5x - 3) = 0$

$(x - 2)(2x - 1)(x + 3) = 0$

$x = -3, \frac{1}{2}, 2$

**7** **a** $p(-1) = 3$

$\therefore\ -b + a + 10 + b = 3$

$a = -7$

**b** $p(\tfrac{1}{3}) = -1$

$\therefore\ \tfrac{1}{27}b - \tfrac{7}{9} - \tfrac{10}{3} + b = -1$

$b - 21 - 90 + 27b = -27$

$b = 3$

**8** **a** $= f(-1) = -1 - 7 - 1 + 10 = 1$

**b** $x^3 - 7x^2 + x + 10 = 1$

$x^3 - 7x^2 + x + 9 = 0$

$x = -1$ is solution $\therefore$ $(x + 1)$ is factor

$$\begin{array}{r|rrrr} & x^2 & -\ 8x & +\ 9 & \\ \hline x+1 & x^3 & -\ 7x^2 & +\ x & +\ 9 \\ & x^3 & +\ x^2 & & \\ \hline & & -\ 8x^2 & +\ x & \\ & & -\ 8x^2 & -\ 8x & \\ \hline & & & 9x & +\ 9 \\ & & & 9x & +\ 9 \\ \hline \end{array}$$

$\therefore\ (x + 1)(x^2 - 8x + 9) = 0$

$x = -1,\ \dfrac{8 \pm \sqrt{64 - 36}}{2} = -1,\ 4 \pm \sqrt{7}$

**9** $f(\tfrac{2}{3}) = 6$

$\therefore\ \tfrac{8}{9} + \tfrac{4}{9}k - \tfrac{14}{3} + 2k = 6$

$8 + 4k - 42 + 18k = 54$

$22k = 88$

$k = 4$

**10** **a** $f(3) = 54 - 63 + 12 - 3 = 0$

$\therefore\ (x - 3)$ is a factor of $f(x)$

**b**

$$\begin{array}{r|rrrr} & 2x^2 & -\ x & +\ 1 & \\ \hline x-3 & 2x^3 & -\ 7x^2 & +\ 4x & -\ 3 \\ & 2x^3 & -\ 6x^2 & & \\ \hline & & -\ x^2 & +\ 4x & \\ & & -\ x^2 & +\ 3x & \\ \hline & & & x & -\ 3 \\ & & & x & -\ 3 \\ \hline \end{array}$$

$\therefore\ f(x) \equiv (x - 3)(2x^2 - x + 1)$

**c** $f(x) = 0 \Rightarrow (x - 3)(2x^2 - x + 1) = 0$

$x = 3$ or $2x^2 - x + 1 = 0$

for $2x^2 - x + 1 = 0$, $b^2 - 4ac = -7$

$b^2 - 4ac < 0 \quad \Rightarrow \quad$ no real roots

$\therefore$ only one real solution

**11** **a** $f(2) = 0$

$\therefore\ 8 + 2p + q = 0$

$q = -2p - 8$

**b** $f(-1) = -15$

$\therefore\ -1 - p + q = -15$

$q = p - 14$

$\therefore\ p - 14 = -2p - 8$

$p = 2,\ q = -12$

**12** $f(-3) = 0$ $\therefore$ $(x + 3)$ is a factor of $f(x)$

$$\begin{array}{r|rrrr} & x^2 & +\ x & -\ 3 & \\ \hline x+3 & x^3 & +\ 4x^2 & +\ 0x & -\ 9 \\ & x^3 & +\ 3x^2 & & \\ \hline & & x^2 & +\ 0x & \\ & & x^2 & +\ 3x & \\ \hline & & & -\ 3x & -\ 9 \\ & & & -\ 3x & -\ 9 \\ \hline \end{array}$$

$\therefore\ f(x) = (x + 3)(x^2 + x - 3)$

other solutions given by $x^2 + x - 3 = 0$

$x = \dfrac{-1 \pm \sqrt{1 + 12}}{2} = \dfrac{-1 \pm \sqrt{13}}{2}$

$x = -2.30$ or $1.30$

**13** **a** $f(-2) = -7$

$\therefore\ (-2 + k)^3 - 8 = -7$

$(k - 2)^3 = 1$

$k = 3$

**b** $f(x) \equiv (x + 3)^3 - 8$

$\therefore\ f(-1) = 2^3 - 8 = 0$

$\therefore\ (x + 1)$ is a factor

**14** **a** $= f(-2) = -8 - 16 + 14 + 8 = -2$

**b** $c = 2$

**c** $g(x) \equiv x^3 - 4x^2 - 7x + 10$

$$\begin{array}{r|l} & x^2 - 6x + 5 \\ \hline x + 2 & x^3 - 4x^2 - 7x + 10 \\ & x^3 + 2x^2 \\ \hline & \quad - 6x^2 - 7x \\ & \quad - 6x^2 - 12x \\ \hline & \qquad 5x + 10 \\ & \qquad 5x + 10 \\ \hline \end{array}$$

$\therefore\ g(x) = (x + 2)(x^2 - 6x + 5)$

$= (x + 2)(x - 1)(x - 5)$

$g(x) = 0 \Rightarrow x = -2, 1, 5$

**15** **a** $f(\tfrac{1}{2}k) = 4$

$\therefore\ \tfrac{1}{8}k^3 - 2k + 1 = 4$

$k^3 - 16k + 8 = 32$

$k^3 - 16k - 24 = 0 \qquad (1)$

**b** $f(-k) = 1$

$\therefore\ -k^3 + 4k + 1 = 1$

$k^3 = 4k$

sub (1) $\Rightarrow\ 4k - 16k - 24 = 0$

$12k = -24$

$k = -2$