1 **a** $f(-2) = 0 \Rightarrow -8 - 20 - 2a + b = 0$
$\Rightarrow -2a + b = 28 \quad (1)$
$f(3) = 0 \Rightarrow 27 - 45 + 3a + b = 0$
$\Rightarrow 3a + b = 18 \quad (2)$
$(2) - (1) \ 5a = -10 = 0 \Rightarrow a = -2$
sub. (1) $\Rightarrow b = 24$

b $f(x) \equiv x^3 - 5x^2 - 2x + 24$
$(x + 2)(x - 3)(ax + b) \equiv x^3 - 5x^2 - 2x + 24$
by inspection
$f(x) \equiv (x + 2)(x - 3)(x - 4)$

2 $f(k) = 8f(\frac{1}{2}k)$
$8k^3 - k^2 + 7 = 8(k^3 - \frac{1}{4}k^2 + 7)$
$8k^3 - k^2 + 7 = 8k^3 - 2k^2 + 56$
$k^2 = 49$
$k = \pm 7$

3 **a** $f(\frac{1}{3}) = \frac{1}{9} - \frac{1}{9} - 4 + 4 = 0$
$\therefore (3x - 1)$ is a factor of $f(x)$

b
$$\begin{array}{r|l} & x^2 + 0x - 4 \\ 3x-1 & 3x^3 - x^2 - 12x + 4 \\ & 3x^3 - x^2 \\ & 0x^2 - 12x \\ & 0x^2 + 0x \\ & -12x + 4 \\ & -12x + 4 \end{array}$$

$\therefore f(x) = (3x - 1)(x^2 - 4)$
$= (3x - 1)(x + 2)(x - 2)$

$f(x) = 0 \Rightarrow (3x - 1)(x + 2)(x - 2) = 0$
$x = -2, \frac{1}{3}$ or 2

4 $6 + 7x - x^3 = 0$
let $f(x) = 6 + 7x - x^3$
$f(1) = 12,\ f(2) = 12,\ f(-1) = 0$
$\therefore (x + 1)$ is a factor of $f(x)$

$$\begin{array}{r|l} & -x^2 + x + 6 \\ x+1 & -x^3 + 0x^2 + 7x + 6 \\ & -x^3 - x^2 \\ & x^2 + 7x \\ & x^2 + x \\ & 6x + 6 \\ & 6x + 6 \end{array}$$

$\therefore (x + 1)(-x^2 + x + 6) = 0$
$-(x + 1)(x - 3)(x + 2) = 0$
$x = -2, -1, 3$
$\therefore (-2, 0), (-1, 0)$ and $(3, 0)$

5 **a** $f(-1) = -4$

$\therefore\ -3 + p - 8 + q = -4$

$p + q = 7 \quad (1)$

$f(2) = 80$

$\therefore\ 24 + 4p + 16 + q = 80$

$4p + q = 40 \quad (2)$

$(2) - (1) \Rightarrow 3p = 33$

$\therefore\ p = 11,\ q = -4$

b $f(x) \equiv 3x^3 + 11x^2 + 8x - 4$

$f(-2) = -24 + 44 - 16 - 4 = 0$

$\therefore\ (x + 2)$ is a factor

c

$$\begin{array}{r|rrrr} & 3x^2 & +5x & -2 & \\ \hline x+2 & 3x^3 & +11x^2 & +8x & -4 \\ & 3x^3 & +6x^2 & & \\ \hline & & 5x^2 & +8x & \\ & & 5x^2 & +10x & \\ \hline & & & -2x & -4 \\ & & & -2x & -4 \\ \hline \end{array}$$

$\therefore\ f(x) = (x + 2)(3x^2 + 5x - 2)$

$= (3x - 1)(x + 2)^2$

$\therefore\ f(x) = 0 \Rightarrow x = -2$ or $\frac{1}{3}$

6 **a** let $f(x) = x^3 - 4x^2 - 7x + 10$

$f(1) = 1 - 4 - 7 + 10 = 0$

$\therefore\ (x - 1)$ is a factor

$$\begin{array}{r|rrrr} & x^2 & -3x & -10 & \\ \hline x-1 & x^3 & -4x^2 & -7x & +10 \\ & x^3 & -x^2 & & \\ \hline & & -3x^2 & -7x & \\ & & -3x^2 & +3x & \\ \hline & & & -10x & +10 \\ & & & -10x & +10 \\ \hline \end{array}$$

$\therefore\ (x - 1)(x^2 - 3x - 10) = 0$

$(x - 1)(x + 2)(x - 5) = 0$

$x = -2,\ 1,\ 5$

b $y^2 = x$ in part **a**

$y^2 = 1,\ 5$ or -2 [no solutions]

$y = \pm 1,\ \pm\sqrt{5}$

7 **a** $= f(-1) = -1 + 7 - 14 + 3 = -5$

b

$$\begin{array}{r|rrrr} & n^2 & +6n & +8 & \\ \hline n+1 & n^3 & +7n^2 & +14n & +3 \\ & n^3 & +n^2 & & \\ \hline & & 6n^2 & +14n & \\ & & 6n^2 & +6n & \\ \hline & & & 8n & +3 \\ & & & 8n & +8 \\ \hline & & & & -5 \end{array}$$

$\therefore\ f(n) = (n + 1)(n^2 + 6n + 8) - 5$

$f(n) = (n + 1)(n + 2)(n + 4) - 5$

c $(n + 1)$ and $(n + 2)$ are consecutive integers

$\therefore$ either $(n + 1)$ or $(n + 2)$ is even

$\therefore\ (n + 1)(n + 2)(n + 4)$ is even

$\therefore\ (n + 1)(n + 2)(n + 4) - 5$ is odd