

MEI Mechanics 1 Projectiles

Section 2: General equations

Solutions to Exercise

1. Horizontally: $x = ut$

$$t = \frac{x}{u}$$

vertically: $y = -\frac{1}{2}gt^2$

$$y = -\frac{g}{2}\left(\frac{x}{u}\right)^2$$

$$y = -\frac{gx^2}{2u^2}$$

(i) If it just clears the fence, $y = -2.5$ when $x = 8$.

$$-2.5 = -\frac{9.8 \times 8^2}{2u^2}$$

$$u^2 = 125.44$$

$$u = 11.2$$

The initial speed is 11.2 ms^{-1} .

(ii) When ball lands, $y = -4.9$

$$-4.9 = -\frac{9.8x^2}{2 \times 11.2^2}$$

$$x^2 = \frac{4.9 \times 2 \times 11.2^2}{9.8}$$

$$x = 11.2$$

It lands 11.2 m horizontally from its starting point so it lands 3.2 m beyond the fence.

2. Vertically: $y = ut \sin \theta - \frac{1}{2}gt^2$

When shell hits ground, $0 = t(210 \sin \theta - 4.9t)$

$$t = \frac{210 \sin \theta}{4.9}$$

Horizontally: $x = ut \cos \theta$

$$= 210 \times \frac{210 \sin \theta}{4.9} \times \cos \theta$$

$$= 9000 \sin \theta \cos \theta$$

$$= 4500 \sin 2\theta$$

(i) At maximum range, angle = 45° .

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$$\begin{aligned}\text{Range} &= 4500 \times \sin 90^\circ \\ &= 4500\end{aligned}$$

The maximum range is 4.5 km.

$$(ii) \quad 3600 = 4500 \sin 2\theta$$

$$\sin 2\theta = 0.8$$

$$2\theta = 53.13^\circ \text{ or } 126.87^\circ$$

$$\theta = 26.6^\circ \text{ or } 63.4^\circ$$

$$3. (i) \text{ Horizontally: } x = ut \cos \theta$$

$$22 = 2u \cos \theta$$

$$u \cos \theta = 11$$

$$\text{Vertically: } y = ut \sin \theta - \frac{1}{2}gt^2$$

$$2.4 = 2u \sin \theta - \frac{1}{2} \times 9.8 \times 4$$

$$u \sin \theta = 11$$

$$\text{Dividing: } \tan \theta = 1 \Rightarrow \theta = 45^\circ$$

$$u = \frac{11}{\sin 45^\circ} = 11\sqrt{2} = 15.6 \text{ ms}^{-1} \text{ (3 s.f.)}$$

It was kicked at 15.6 ms^{-1} at 45° to the horizontal.

$$(ii) \text{ At maximum height: } v_y^2 = u_y^2 - 2gs$$

$$0 = (11\sqrt{2} \sin 45^\circ)^2 - 2 \times 9.8s$$

$$s = \frac{121}{19.6} = 6.17 \text{ m (3 s.f.)}$$

$$(iii) \text{ When ball lands: } y = ut \sin \theta - \frac{1}{2}gt^2$$

$$0 = t(11 - 4.9t)$$

$$t = \frac{11}{4.9}$$

$$x = ut \cos \theta = 11t = 11 \times \frac{11}{4.9} = 24.7$$

The ball lands 24.7 m from O (3 s.f.)

$$4. (i) \text{ At greatest height: } v_y^2 = u_y^2 - 2gs$$

$$0 = (25 \sin 50^\circ)^2 - 2 \times 9.8s$$

$$s = \frac{(25 \sin 50^\circ)^2}{19.6} = 18.7 \text{ m (3 s.f.)}$$

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(ii) When $x = 50$, $50 = 25t \cos 50^\circ$

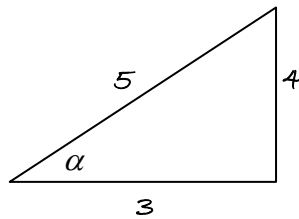
$$t = \frac{2}{\cos 50^\circ}$$

At this time, $y = 25t \sin 50^\circ - \frac{1}{2} \times 9.8t^2$

$$\begin{aligned} &= 25 \sin 50^\circ \times \frac{2}{\cos 50^\circ} - 4.9 \left(\frac{2}{\cos 50^\circ} \right)^2 \\ &= 12.15 \end{aligned}$$

The ball will clear the pavilion.

5. (i)



$$y = 60t \sin \alpha - \frac{1}{2}gt^2$$

$$99 = 60t \times \frac{4}{5} - 5t^2$$

$$5t^2 - 48t + 99 = 0$$

$$(5t - 33)(t - 3) = 0$$

$$t = 3 \text{ or } t = 6.6$$

The particle is at a height of 99 m at 3 seconds and at 6.6 seconds.

(ii) $x = 60t \cos \alpha = 60t \times \frac{3}{5} = 36t$

When $t = 3$, $x = 36 \times 3 = 108$

When $t = 6.6$, $x = 36 \times 6.6 = 237.6$

When the particle is at a height of 99 m, it is at a horizontal distance of 108 m or 237.6 m from O.