C1 DIFFERENTIATION Worksheet F

1 $$f(x) \equiv 7 + 24x + 3x^2 - x^3.$$

a Find $f'(x)$. **(2)**

b Find the set of values of x for which $f(x)$ is increasing. **(4)**

2 A curve has the equation $y = 4x^3 + 9x^2 - 12x - 2$.

a Find the coordinates of the stationary points of the curve. **(6)**

b Determine whether each stationary point is a maximum or a minimum point. **(3)**

3 Differentiate $x^2 + \frac{1}{2x}$ with respect to x. **(3)**

4 $$f(x) = (x + 2)(x - 1)^2.$$

a Sketch the curve $y = f(x)$, showing the coordinates of any points where the curve meets the coordinate axes. **(3)**

b Find $f'(x)$. **(4)**

c Find the equation of the tangent to the curve $y = f(x)$ at the point where it crosses the y-axis, giving your answer in the form $y = mx + c$. **(3)**

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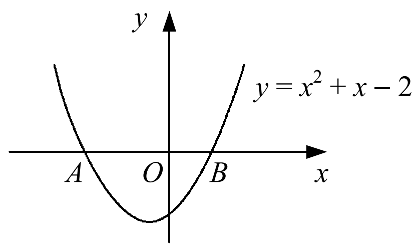

The diagram shows the curve $y = x^2 + x - 2$. The curve crosses the x-axis at the points A $(a, 0)$ and B $(b, 0)$ where $a < b$.

a Find the values of a and b. **(3)**

b Show that the normal to the curve at A has the equation

$$x - 3y + 2 = 0.$$ **(5)**

The tangent to the curve at B meets the normal to the curve at A at the point C.

c Find the exact coordinates of C. **(4)**

6 $$y = x^2 + 3x^{\frac{1}{2}}.$$

a Find $\frac{dy}{dx}$. **(2)**

b Show that $2x\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6x = 0$. **(4)**

7 A curve has the equation $y = 2 + \frac{4}{x}$.

a Find an equation of the normal to the curve at the point M $(4, 3)$. **(5)**

The normal to the curve at M intersects the curve again at the point N.

b Find the coordinates of the point N. **(5)**

8 A curve has the equation $y = 2x^2 - 7x + 1$ and the point A on the curve has x-coordinate 2.

a Find an equation of the tangent to the curve at A. **(4)**

The normal to the curve at the point B is parallel to the tangent at A.

b Find the coordinates of B. **(3)**

9 $$\mathrm{f}(x) \equiv x^2 + \frac{16}{x}, \quad x \neq 0.$$

a Find $\mathrm{f}'(x)$. **(2)**

b Find the coordinates of the turning point of the curve $y = \mathrm{f}(x)$ and determine its nature. **(6)**

10 The curve C has the equation $y = x^3 - x^2 + 2x - 4$.

a Find an equation of the tangent to C at the point $(1, -2)$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(5)**

b Prove that the curve C has no stationary points. **(4)**

11 The curve C has the equation $y = x - 3x^{\frac{1}{2}} + 3$ and passes through the point P (4, 1).

a Show that the tangent to C at P passes through the origin. **(5)**

The normal to C at P crosses the y-axis at the point Q.

b Find the area of triangle OPQ, where O is the origin. **(4)**

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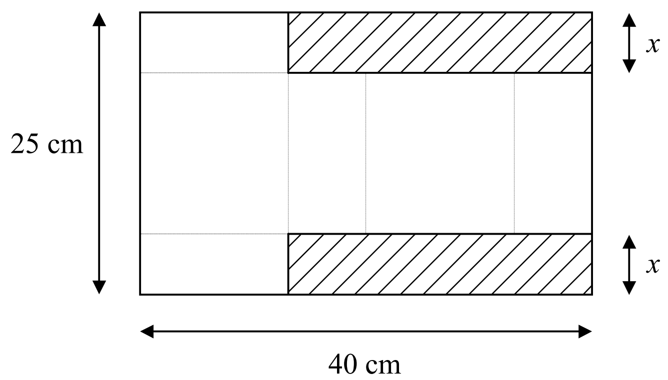

Two identical rectangles of width x cm are removed from a rectangular piece of card measuring 25 cm by 40 cm as shown in the diagram above. The remaining card is the net of a cuboid of height x cm.

a Find expressions in terms of x for the length and width of the base of the cuboid formed from the net. **(3)**

b Show that the volume of the cuboid is $(2x^3 - 65x^2 + 500x)$ cm^3. **(2)**

c Find the value of x for which the volume of the cuboid is a maximum. **(5)**

d Find the maximum volume of the cuboid and show that it is a maximum. **(3)**

13 **a** Find the coordinates of the stationary points on the curve

$$y = 2 + 9x + 3x^2 - x^3.$$ **(6)**

b Determine whether each stationary point is a maximum or minimum point. **(3)**

c State the set of values of k for which the equation

$$2 + 9x + 3x^2 - x^3 = k$$

has three solutions. **(2)**